Unconventional pairing in three-dimensional topological insulators with warped surface state

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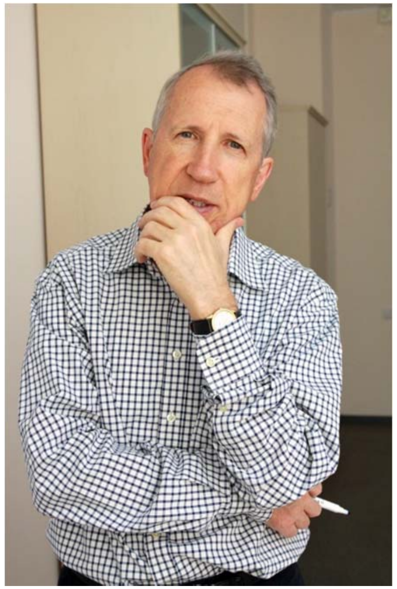


DIPC

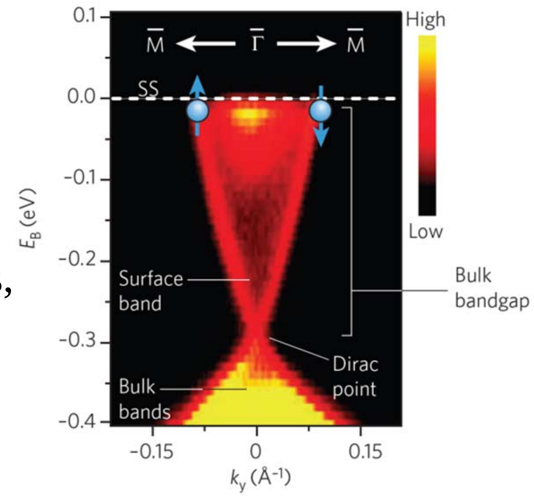
Topological insulator in a nutshell

…a new state of matter that has been predicted and discovered!

- Bulk is insulating; edge (2D)/ surface (3D) a very good conductor.
- Important ingredient: spin-orbit coupling:

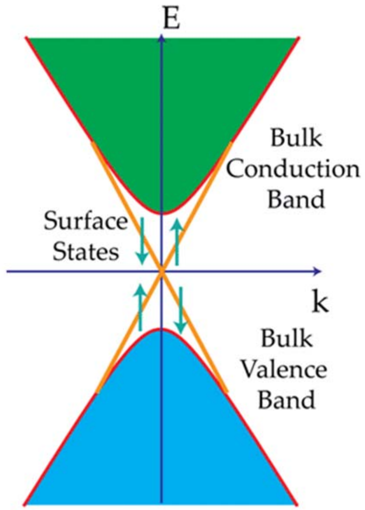
opposite force for opposite spins.

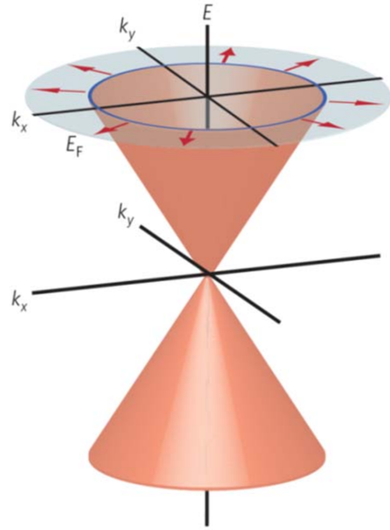
- Topological invariant is insensitive to any continuous deformation of Hamiltonian (topological protection): disorder, geometry, weak interactions, etc…



Examples:

- 2D: HgTe/CdTe; 3D: Bi_2Se_3, Bi_2Te_3 Sb_2Te_3, $TlBiSe_2$, Bi_2Te_2Se.





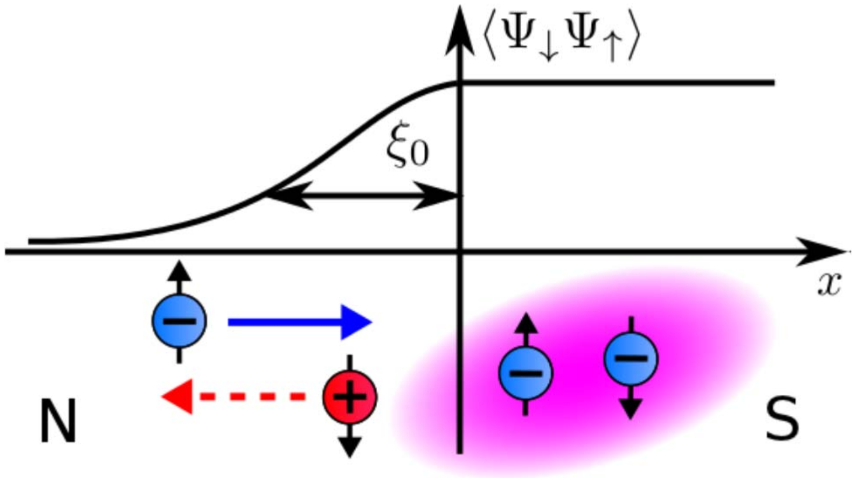
2D

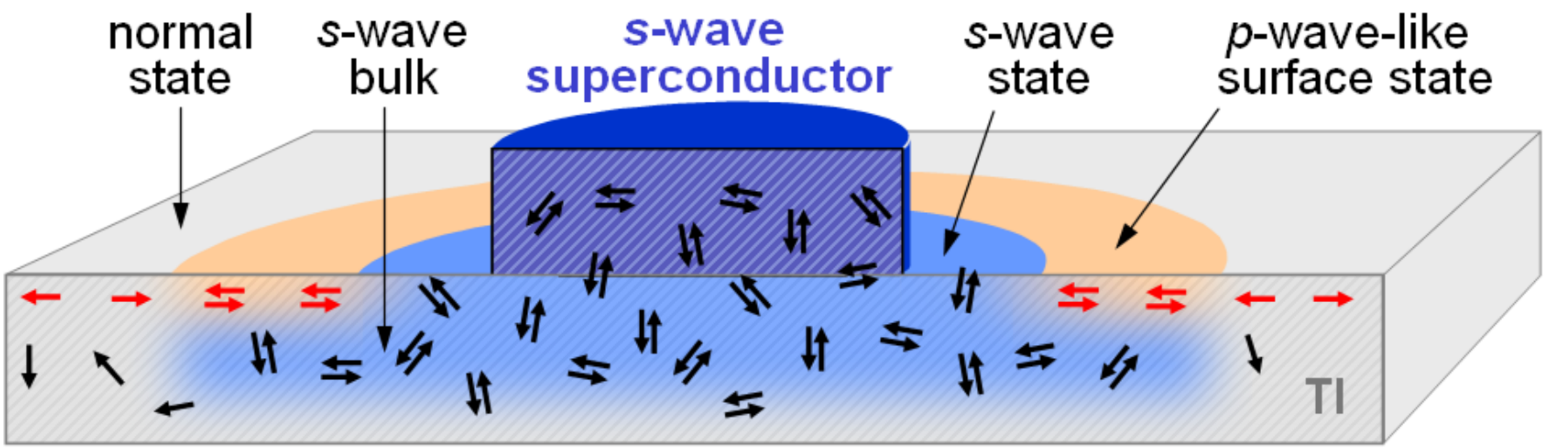
Theo1: *C.L. Kane and E.J. Mele, **PRL** 95, 226801 (2005)*
Theo2: *B.A. Bernevig et al., **Science** 314, 1757 (2006)*
Exp: *M. König et al., **Science** 318, 766 (2007)*

3D

Theo: *L. Fu, et al., **PRL** 98, 106803 (2007)*
Exp1: *Zhang H. et al., **Nat. Phys.** 5, 438 (2009)*
Exp3: *S. Takafumi et al., **PRL** 105, 136802 (2010)*

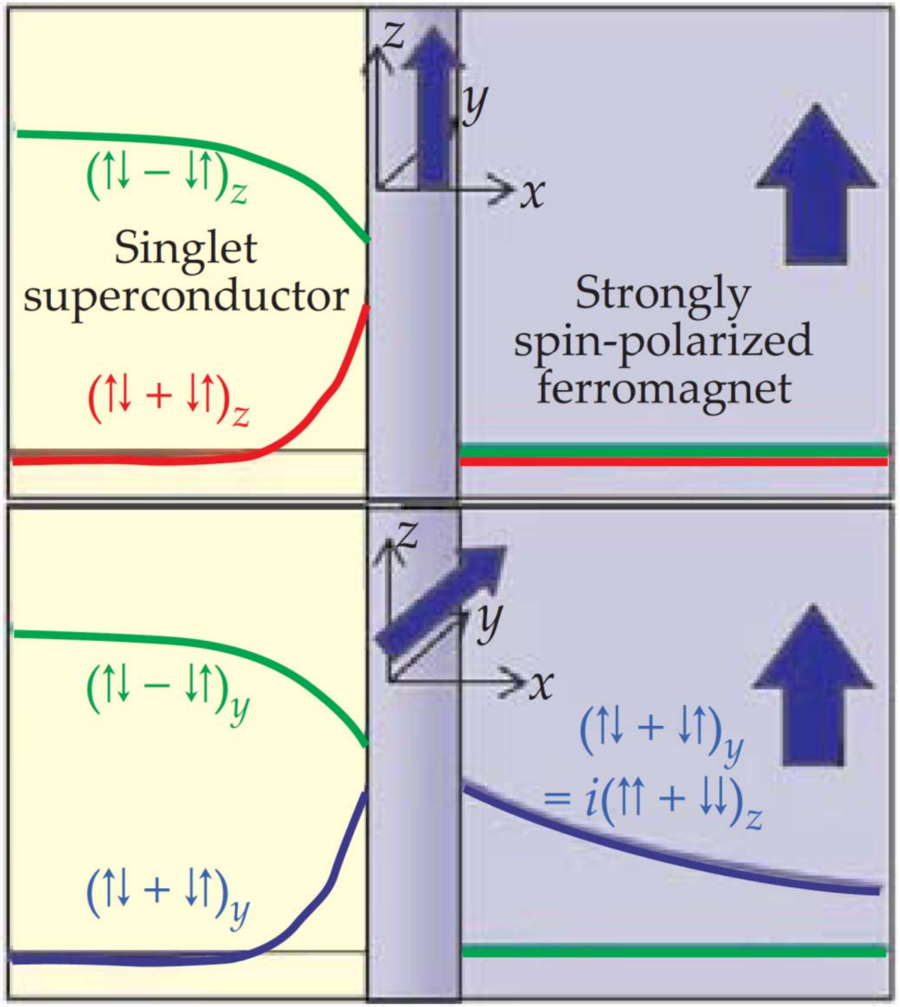
Superconductor/ topological insulator proximity effect





J. Shen et al., arXiv:1303.5598 (2013)

Spin-triplet superconductivity



Hetero-spin triplet component

$$(\uparrow\downarrow + \downarrow\uparrow)$$

Equal-spin triplet components

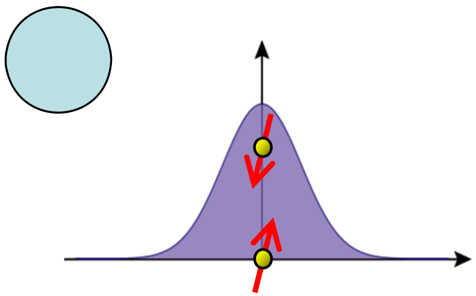
$$(\uparrow\uparrow - \downarrow\downarrow)$$

$$(\uparrow\uparrow + \downarrow\downarrow)$$

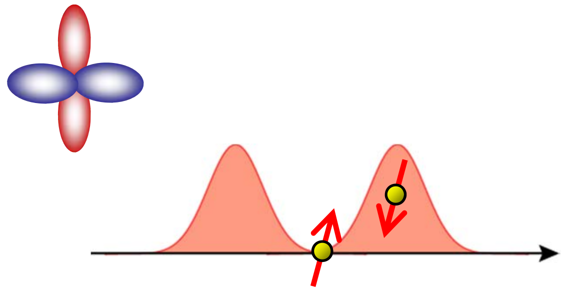
M. Eschrig, Physics Today (2011)

Conventional classification of the pairing symmetry

Spin-singlet Cooper pair → Even Parity

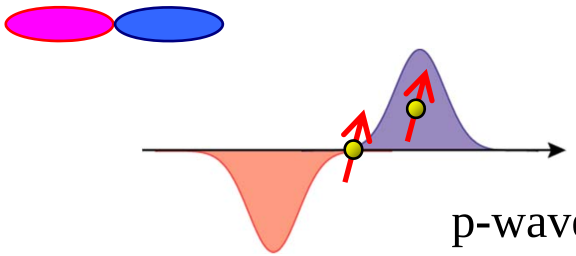


s-wave **BCS**



d-wave **Cuprate**

Spin-triplet Cooper pair → Odd Parity

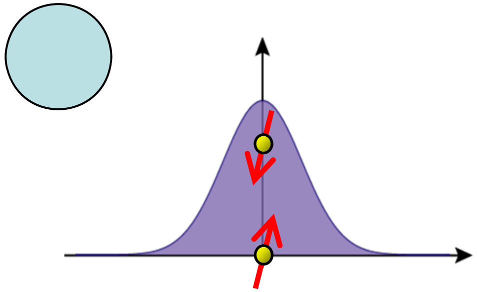


p-wave

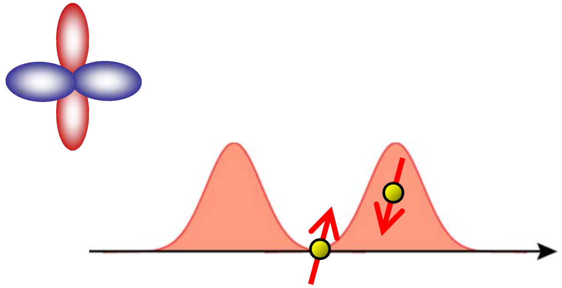
In both cases, the pair amplitude is an even function of energy (or Matsubara frequency).

Conventional classification of the pairing symmetry

Spin-singlet Cooper pair → Even Parity

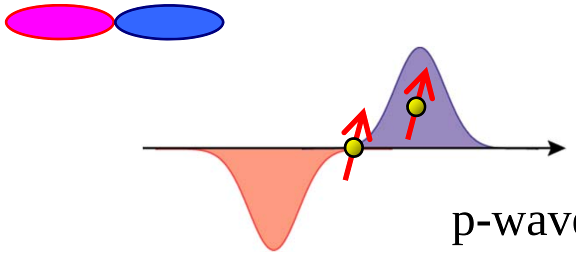


s-wave **BCS**



d-wave **Cuprate**

Spin-triplet Cooper pair → Odd Parity



p-wave **^{3}He Sr_2RuO_4 UPt_3**

However, the so-called odd-frequency pairing states when the pair amplitude is an odd function of energy can also exist.

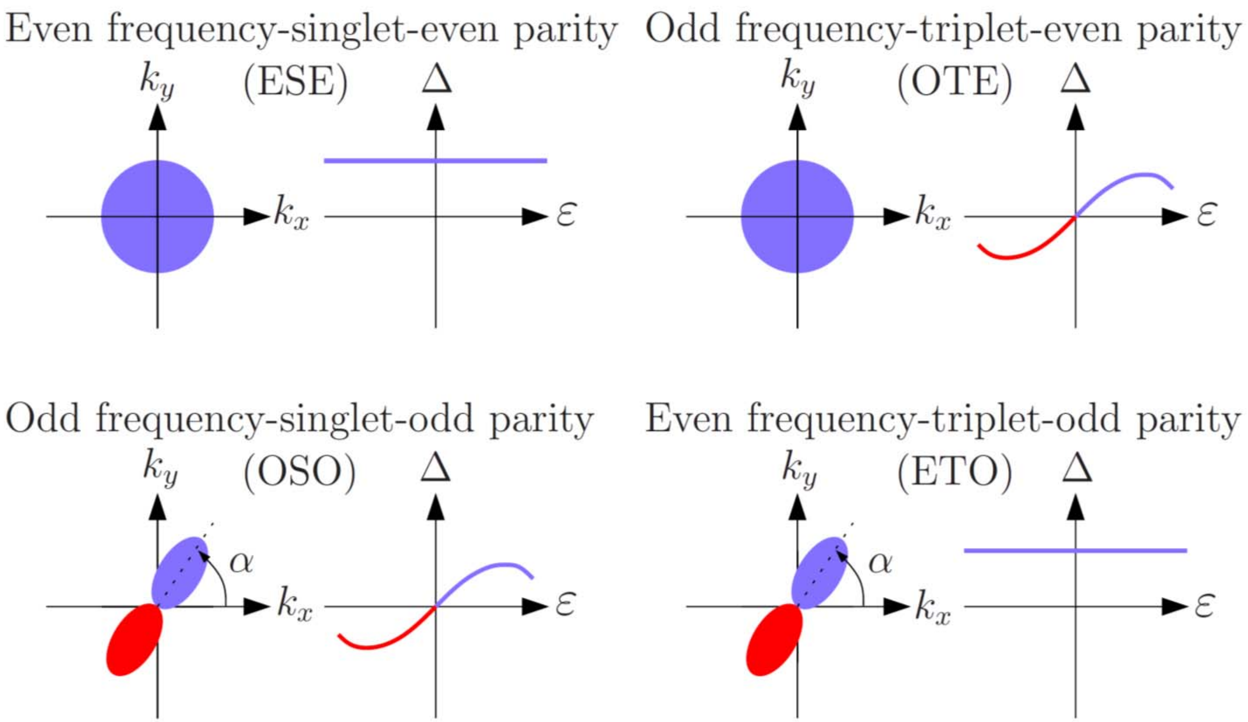
Symmetry classification of induced pair potential

Fermi-Dirac statistics

Symmetry of pair wave functions:

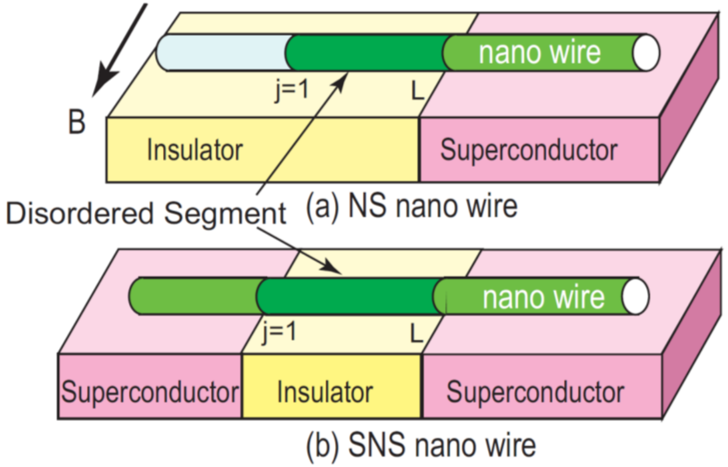
$$\mathbf{k} \otimes \sigma \otimes \omega = \mathrm{odd}$$

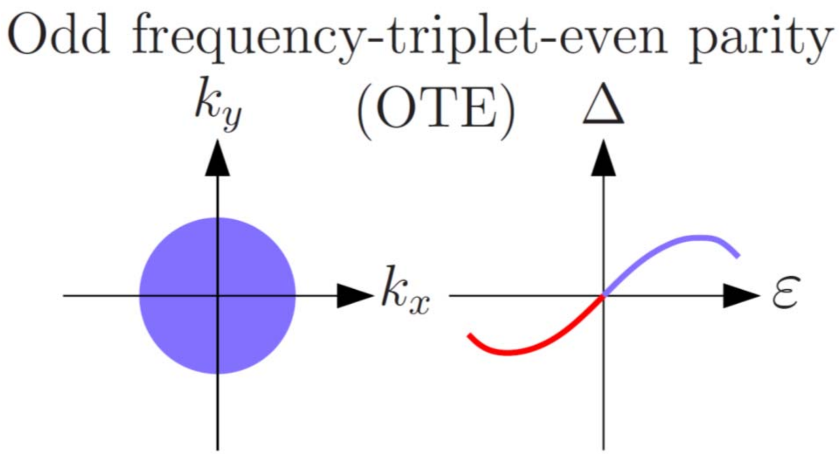
	$F_{\sigma\sigma'}(\omega,k) = -F_{\sigma'\sigma}(-\omega,-k)$		
	$\omega \to -\omega$	$\sigma \leftrightarrow \sigma'$	$k \to -k$
ESE	$+$	$-$	$+$
OSO	$-$	$-$	$-$
ETO	$+$	$+$	$-$
OTE	$-$	$+$	$+$



J. Linder et al., PRB (2008)

Odd-frequency pairing and Majorana state





Asano, Tanaka, PRB (2013)

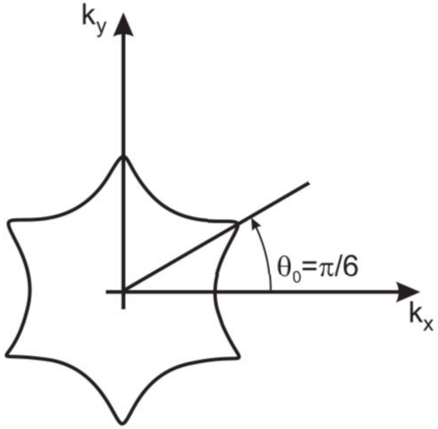
The physics behind the anomalous transport can be understood in terms of the odd-frequency Cooper pairing. We conclude that Majorana fermions and odd-frequency Cooper pairs in solids are two sides of a same coin.

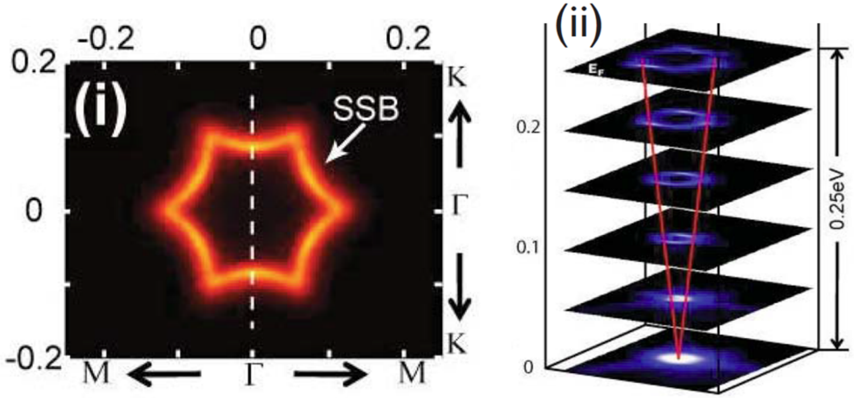
Hexagonal warping in 3D Topological insulators

$$\hat{H}(\mathbf{k}) = -\mu + v(k_x\hat{\sigma}_y - k_y\hat{\sigma}_x) + \hat{H}_{\mathrm{w}}(\mathbf{k})$$

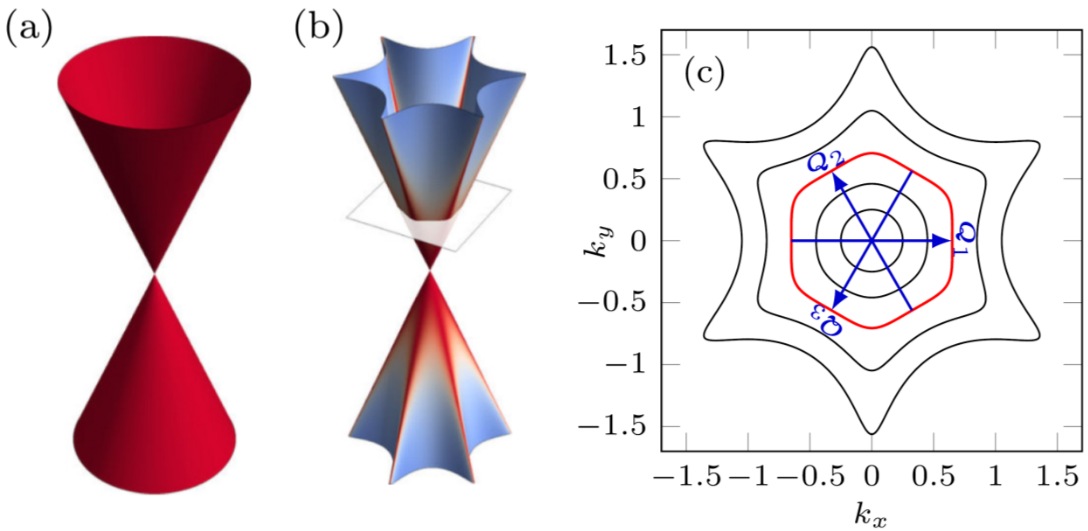
$$\hat{H}_{\mathrm{w}}(\mathbf{k}) = \lambda k^3 \cos(3\theta)\hat{\sigma}_z$$

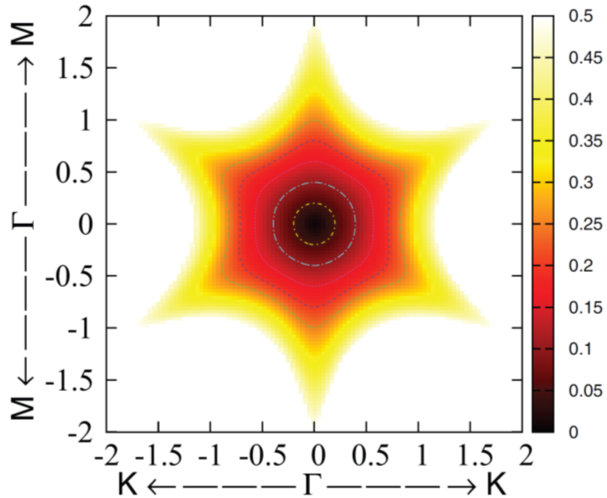
Fu, PRL (2009)





Chen et al., Science (2009)





Mendle, Kotetes, Schon, PRB (2015)

Li, Carbotte, PRB (2013)

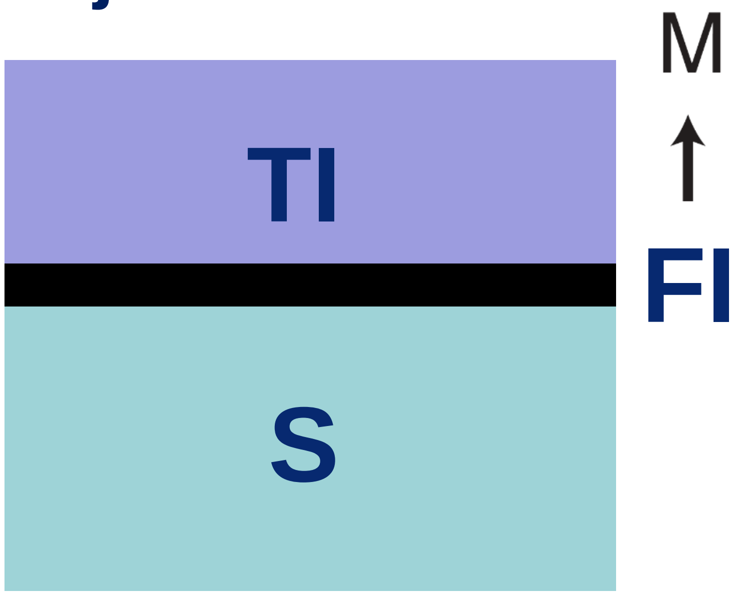
Model: S/ FI/ TI hybrid junction

Bogoliubov – de Gennes – Dirac Hamiltonain

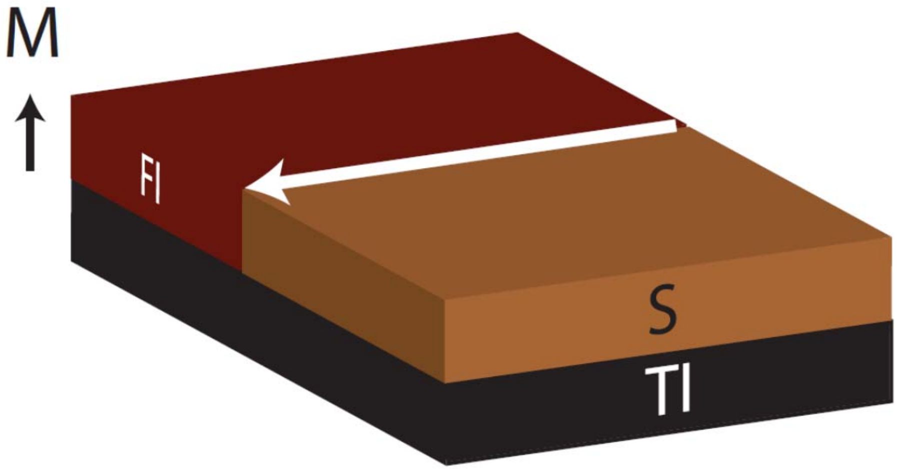
$$\check{H}_S(\mathbf{k}) = \begin{pmatrix} \hat{H}(\mathbf{k}) + M\hat{\sigma}_z & \hat{\Delta} \\ -\hat{\Delta} & -\hat{H}^*(-\mathbf{k}) - M\hat{\sigma}_z \end{pmatrix}$$

Green's function (Nambu + spin space)

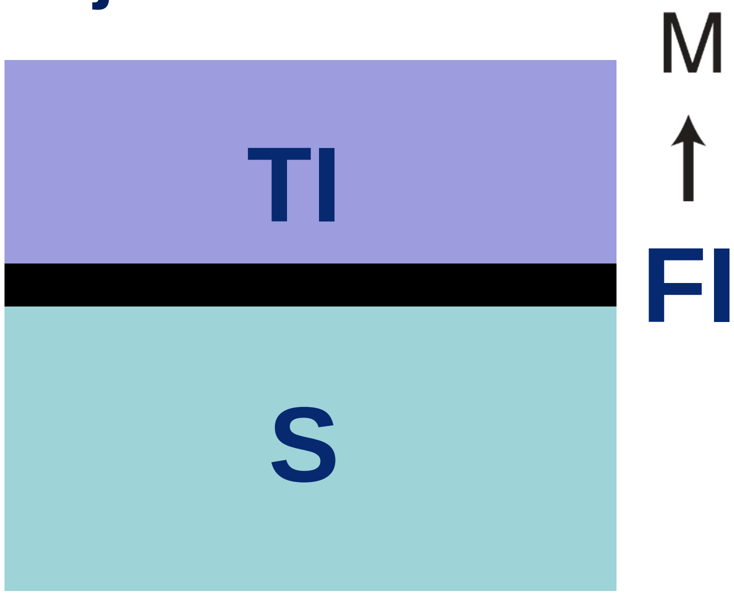
$$\left[E - \check{H}_S(\mathbf{k})\right]\check{G} = \check{1} \qquad \check{G} = \begin{pmatrix} \hat{G}_{\mathrm{ee}} & \hat{G}_{\mathrm{eh}} \\ \hat{G}_{\mathrm{he}} & \hat{G}_{\mathrm{hh}} \end{pmatrix}$$



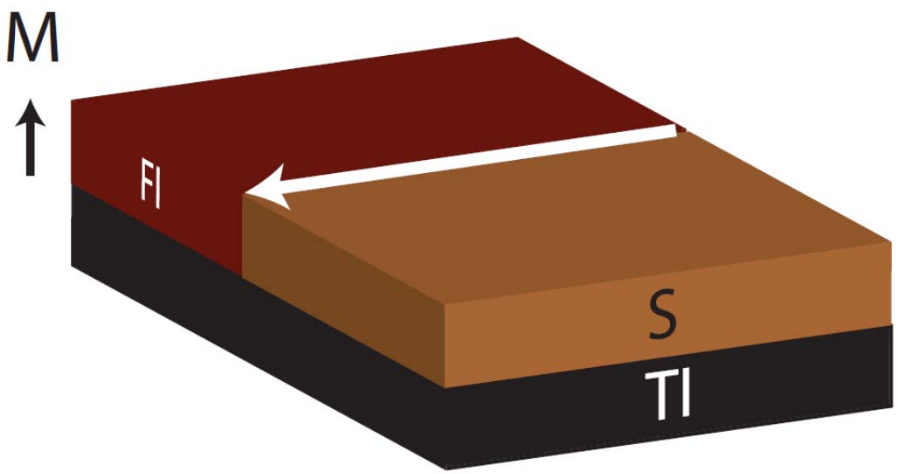
Model: S/ FI/ TI hybrid junction

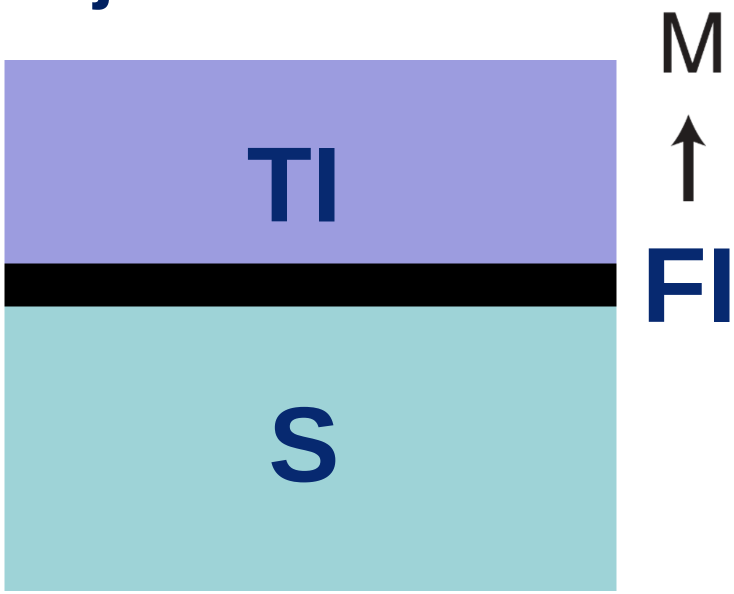


Tanaka, Yokoyama, Nagaosa, PRL (2008)



Model: S/ FI/ TI hybrid junction





Tanaka, Yokoyama, Nagaosa, PRL (2008)

Anomalous Green's function

Bergeret, Volkov, Efetov, RMP (2005)

$$\hat{G}_{\text{eh}} = i\left(f_0\hat{\sigma}_0 + f_x\hat{\sigma}_x + f_y\hat{\sigma}_y + f_z\hat{\sigma}_z\right)\hat{\tau}_y$$

$(\uparrow\downarrow - \downarrow\uparrow)$ singlet   triplet   triplet   triplet $(\uparrow\downarrow + \downarrow\uparrow)$

$(\uparrow\uparrow - \downarrow\downarrow)$ $(\uparrow\uparrow + \downarrow\downarrow)$

No warping

$$\check{H}_S(\mathbf{k}) = \begin{pmatrix} \hat{H}(\mathbf{k}) + M\hat{\sigma}_z & \hat{\Delta} \\ -\hat{\Delta} & -\hat{H}^*(-\mathbf{k}) - M\hat{\sigma}_z \end{pmatrix}$$

$$\hat{H}(\mathbf{k}) = -\mu + v(k_x\hat{\sigma}_y - k_y\hat{\sigma}_x)$$

$$\hat{G}_{\mathrm{eh}} = i\left(f_0\hat{\sigma}_0 + f_x\hat{\sigma}_x + f_y\hat{\sigma}_y + f_z\hat{\sigma}_z\right)\hat{\tau}_y$$

Anomalous Green's function symmetry, Z is even in E and k

$$f_0 = \frac{\Delta}{Z}\left(E^2 + M^2 - \mu^2 - \Delta^2 - v^2k^2\right), \quad (\uparrow\downarrow - \downarrow\uparrow)$$ **ESE**

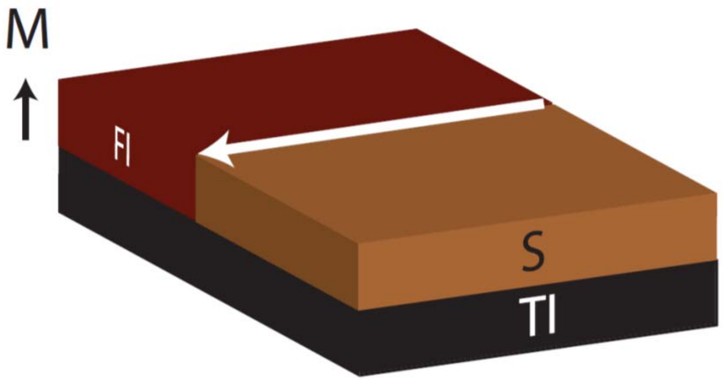
$$f_x = \frac{2\Delta}{Z}kv\left[\mu\sin(\theta) + iM\cos(\theta)\right], \quad (\uparrow\uparrow - \downarrow\downarrow)$$ **ETO**

$$f_y = -\frac{2\Delta}{Z}kv\left[\mu\cos(\theta) - iM\sin(\theta)\right], \quad (\uparrow\uparrow + \downarrow\downarrow)$$ **ETO**

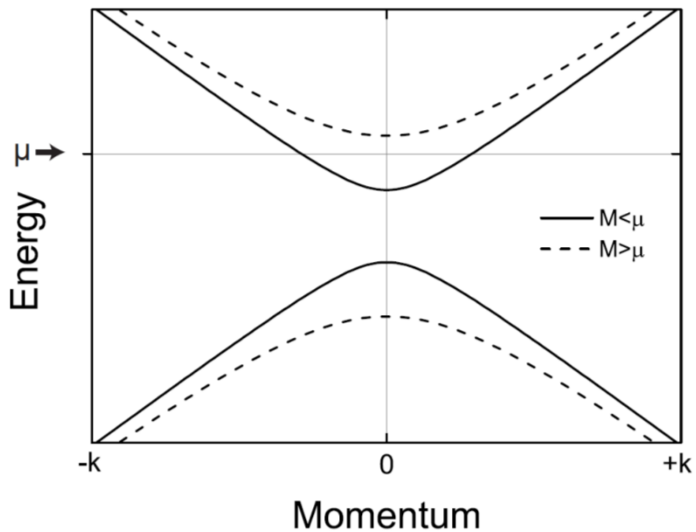
$$f_z = \frac{2\Delta}{Z}EM. \quad (\uparrow\downarrow + \downarrow\uparrow)$$ **OTE** **Majorana mode**

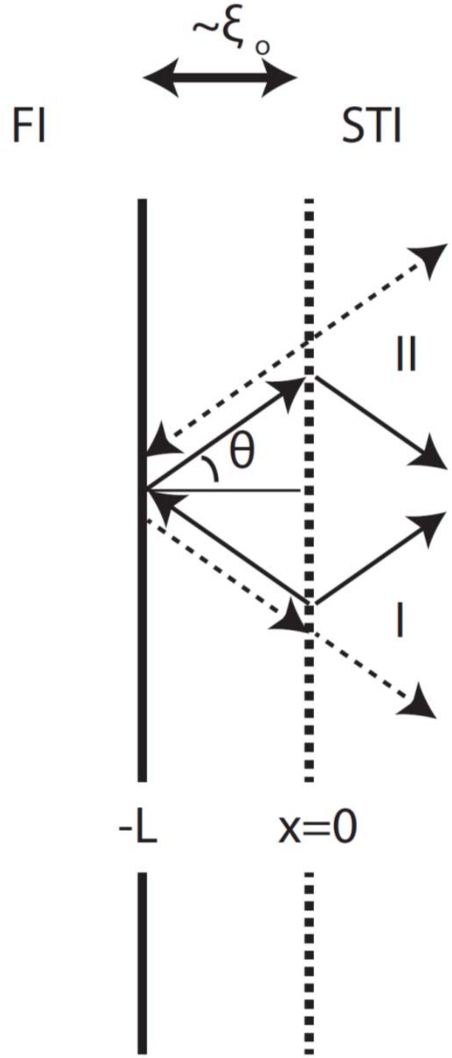
Vasenko, Golubov, Silkin, Chulkov, JETP Lett. (2017)

Majorana fermion realization



$$\frac{E}{\Delta} = -\sin(\theta)$$
$$= -k_y/|k|.$$





Snelder, Golubov, Asano, Brinkman, J. Phys.: Cond. Mat. (2015)

Finite warping

$$\check{H}_S(\mathbf{k}) = \begin{pmatrix} \hat{H}(\mathbf{k}) + M\hat{\sigma}_z & \hat{\Delta} \\ -\hat{\Delta} & -\hat{H}^*(-\mathbf{k}) - M\hat{\sigma}_z \end{pmatrix}$$

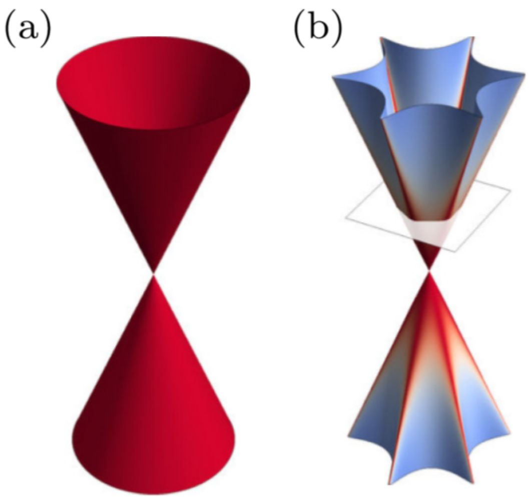
$$\hat{H}(\mathbf{k}) = -\mu + v(k_x\hat{\sigma}_y - k_y\hat{\sigma}_x) + \hat{H}_{\mathrm{w}}(\mathbf{k})$$

$$\hat{G}_{\mathrm{eh}} = i\left(f_0\hat{\sigma}_0 + f_x\hat{\sigma}_x + f_y\hat{\sigma}_y + f_z\hat{\sigma}_z\right)\hat{\tau}_y \qquad f_i = f_i^+ + f_i^-$$

Spin-singlet component $(\uparrow\downarrow - \downarrow\uparrow)$

$$f_0^+ = \left(E^2 + M^2 - \mu^2 - \Delta^2 - E_S^2\right)F_{\mathrm{even}}/2, \quad \textbf{ESE}$$

$$f_0^- = \left(E^2 + M^2 - \mu^2 - \Delta^2 - E_S^2\right)F_{\mathrm{odd}}/2. \quad \textbf{OSO}$$



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$$\hat{H}(\mathbf{k}) = -\mu + v(k_x\hat{\sigma}_y - k_y\hat{\sigma}_x) + \hat{H}_{\mathrm{w}}(\mathbf{k})$$

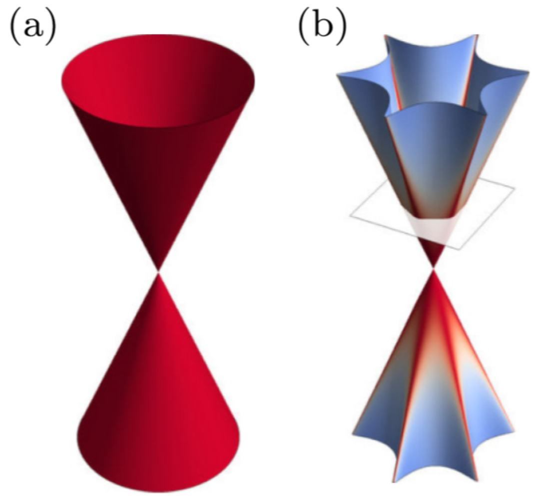
Equal spin triplet components $(\uparrow\uparrow - \downarrow\downarrow)$ $(\uparrow\uparrow + \downarrow\downarrow)$

$$f_x^+ = kv[\mu\sin(\theta) + iM\cos(\theta)]F_{\mathrm{even}},$$ **ETO**

$$f_x^- = kv[\mu\sin(\theta) + iM\cos(\theta)]F_{\mathrm{odd}},$$ **OTE**

$$f_y^+ = -kv[\mu\cos(\theta) - iM\sin(\theta)]F_{\mathrm{even}},$$ **ETO**

$$f_y^- = -kv[\mu\cos(\theta) - iM\sin(\theta)]F_{\mathrm{odd}}.$$ **OTE**



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$$\hat{H}(\mathbf{k}) = -\mu + v(k_x\hat{\sigma}_y - k_y\hat{\sigma}_x) + \hat{H}_w(\mathbf{k})$$

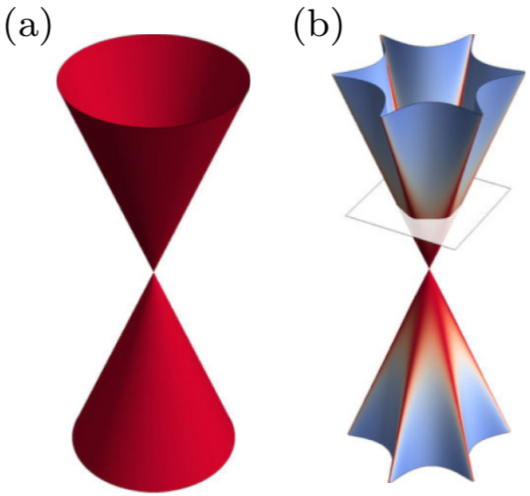
Hetero-spin triplet component $(\uparrow\downarrow + \downarrow\uparrow)$

$$f_z = f_z^- + f_z^+,$$

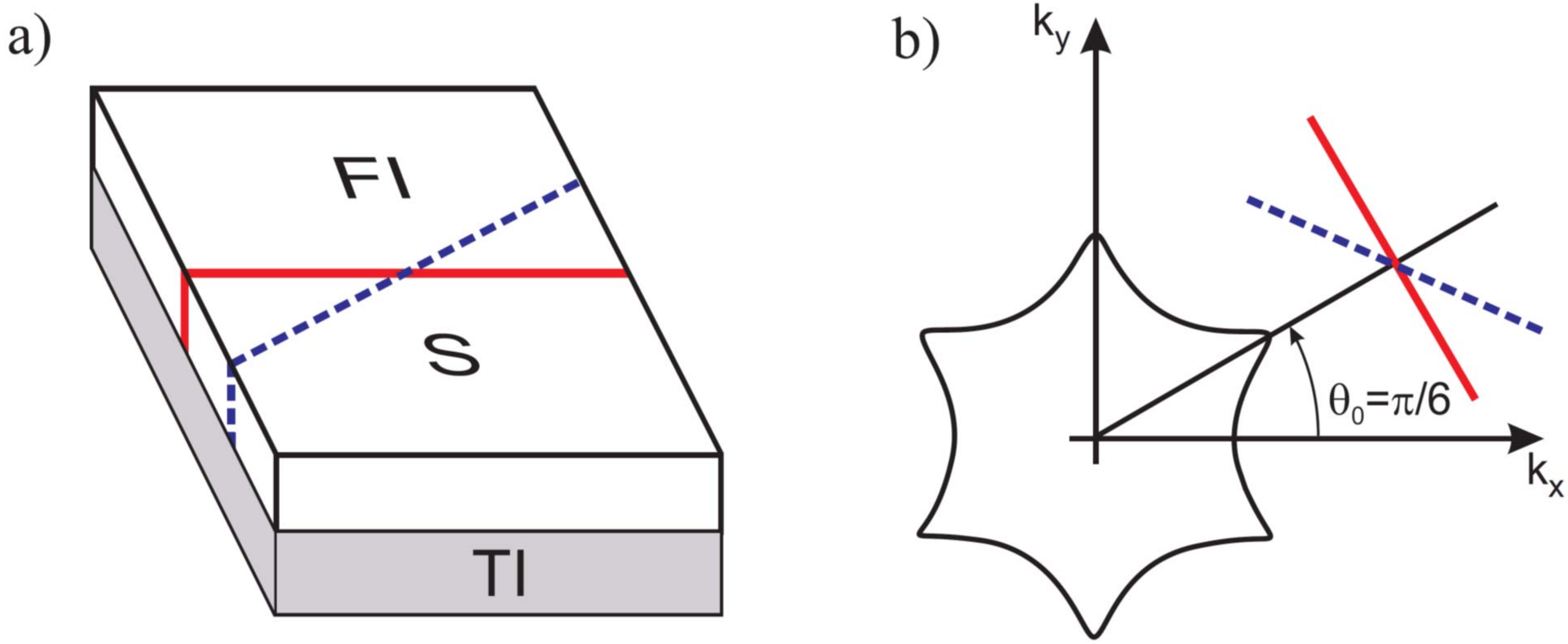
$$f_z^- = EMF_{\text{even}} - \mu\lambda k^3\cos(3\theta)F_{\text{odd}},$$ **OTE**

$$f_z^+ = EMF_{\text{odd}} - \mu\lambda k^3\cos(3\theta)F_{\text{even}}.$$ **ETO**

$$\theta_n = \pi/6 + \pi n/3$$



Majorana fermion (?) and warping

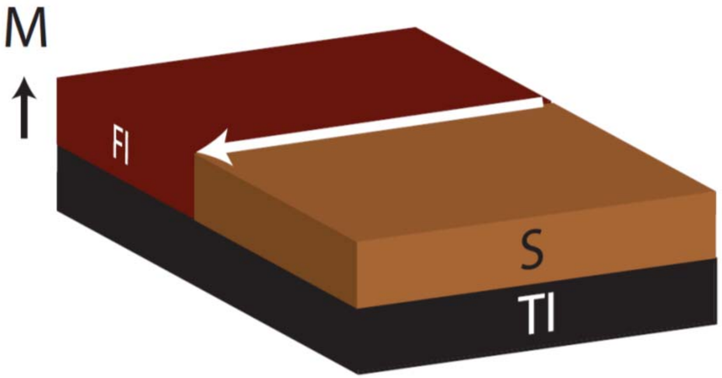


Vasenko, Golubov, Silkin, Chulkov, J. Phys.: Cond. Matt. (2017)

$$\theta_n = \pi/6 + \pi n/3$$

Spontaneous supercurrent

$$\hat{H}_M(\mathbf{k}) = -\mu + v(k_x\hat{\sigma}_y - k_y\hat{\sigma}_x) + \lambda k^3\cos(3\theta)\hat{\sigma}_z + M\hat{\sigma}_z$$



Let us project this Hamiltonian on the S/FI interface, i.e., on the y axis. Then the effective one-dimensional Hamiltonian for electronic states at the S/FI interface will look like ($k_x \sim 0$),

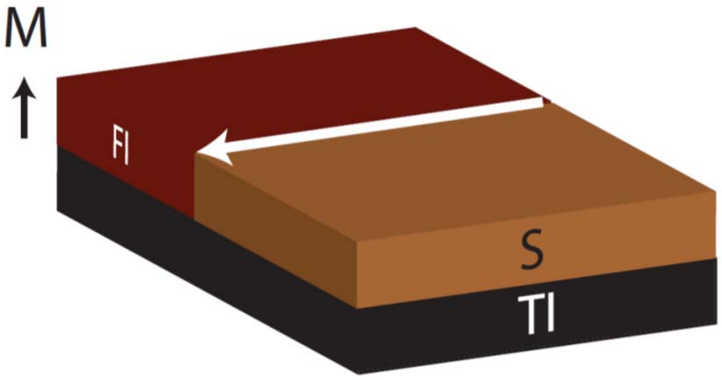
$$\hat{H}_{\text{eff}}(k_y) = -\mu - vk_y\hat{\sigma}_x + \hat{\sigma}_z\lambda k_y^3\cos(3\theta) + \hat{\sigma}_z M$$

From the viewpoint of the time reversal and spatial symmetries, it is equivalent to the following one dimensional Hamiltonian of a topological nanowire,

$$\boxed{\hat{H}(\mathbf{k}) = -\mu + vk_x\hat{\sigma}_y + \hat{\sigma}_x M_x + \hat{\sigma}_y M_y + \hat{\sigma}_z M_z}$$

Spontaneous supercurrent

$$\hat{H}_M(\mathbf{k}) = -\mu + v(k_x\hat{\sigma}_y - k_y\hat{\sigma}_x) + \lambda k^3\cos(3\theta)\hat{\sigma}_z + M\hat{\sigma}_z$$



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Spontaneous supercurrent at zero phase difference.

Nesterov, Houzet, Meyer, PRB (2016)

Review

- We discuss singlet to triplet mixing in proximized 3D topological insulators with warped surface state
- We speculate on the selection rule for Majorana Fermion realization in S/FI structures formed on the surface of the TI: S/FI boundary should be properly aligned with respect to the snowflake contour.
- Spontaneous currents in S/TI hybrids at nonzero warping.

Thank you!