52nd annual meeting of the society for mathematical psychology

17th international conference on cognitive modeling

program & abstracts

Palais des Congrès, Montréal

A nonparametric baseline model for conducting model checking and model comparison in one step

Author(s): Cox, Gregory Edward; Annis, Jeffrey (Vanderbilt University, United States of America). *Contact:* `gregcox7@gmail.com`. *Abstract:* Cognitive models are often too complex to be compared qualitatively, making quantitative model comparison an essential part of mathematical psychology. Bayes factors are a powerful and popular method for quantitative model comparison, but they only indicate the relative support among a set of models and cannot, on their own, assess the absolute quality of a model. Model checking is typically limited to graphical inspection or comparison with summary statistics and is divorced from model comparison. As a step toward unification of model checking and model comparison, we propose a nonparametric "reference" model that serves as a baseline in Bayesian model comparison. This reference model involves ideas from bootstrapping and kernel density estimation, treating the probability concentrated on each observation and the width of the region over which it is distributed as stochastic. The result is a model that assigns likelihoods to each observation but that does not incorporate any information/assumptions beyond that the data-generating distribution resembles the observed data. Any model that performs at least as well as this reference model therefore captures structure in the data and should be considered a viable candidate, such that its victory over another viable model is meaningful. The reference model is easily "plugged in" as a candidate in any likelihood-based model comparison, revealing the viability of the set of models under consideration. We illustrate the utility of this reference model in a set of toy examples as well as in a case of comparing different response time models.

Modeling (Salons 6 & 7)

Mathematical psychology in the wild - why and how? Insights from applying basic modelling concepts to applied problems in traffic safety and self-driving cars

Author(s): Markkula, Gustav (University of Leeds, United Kingdom). *Contact:* `g.markkula@leeds.ac.uk`.
Abstract: Mathematical models of human perception, cognition, and behaviour provide an essential means of stringent knowledge-building in the psychological and cognitive sciences. However, these models also hold large potential value as tools in more applied contexts. What does it take to bring models out of the science lab, over to real applications, and how might this benefit both society and the involved researchers themselves? In this talk, I will first provide an overview of work by myself and collaborators on mathematical modelling of road user behaviour, with applications in traffic safety and vehicle automation. I will describe how a number of open applied questions in this domain have been mapped to existing basic scientific knowledge, including models of evidence accumulation (drift diffusion), predictive coding, and action intent recognition. I will present recent, not yet published results from this line of work, showing how especially accumulator models can be leveraged$^{(1)}$ in combination with predictive coding ideas to predict human responses to vehicle automation failures,$^{(2)}$ with EEG data to provide further insight into human decision making in traffic emergencies, and$^{(3)}$ to model the complex interplay of human (or automated) road users negotiating for space in traffic. In the second part of the talk, I will provide a more general discussion on the topic of transforming basic models into applied ones, how to go about it, and how it can lead to not only societal impact and increased research funding, but also to novel insights and advances in the basic sciences.

Making decisions on intransitivity of superiority: is a general normative model possible?

Author(s): Poddiakov, Alexander (National Research University Higher School of Economics, Russian Federation). *Contact:* `apoddiakov@gmail.com`. *Abstract:* The transitivity axiom (if A is superior to B, and B is superior to C then A is superior to C) often leads people to infer that A is superior to C in all cases. Yet some areas with objective intransitivity of superiority (A beats B, B beats C yet C beats A) are known: intransitive sets

of math objects (dice, lotteries in intransitive relations “stochastically greater than”), intransitive competition in biology, etc. All these intransitive relations are probabilistic. We have designed objects in deterministic intransitive relations. Intransitive machines demonstrate unexpected intransitivity in relations “to rotate faster”, “to be stronger”, etc. in some geometrical constructions - Condorcet-like compositions. Intransitive chess positions are such that Position A for White is preferable to Position B for Black (i.e., when offered a choice, one should choose A), Position B for Black is preferable to Position C for White, which is preferable to Position D (Black) – but the latter is preferable to Position A. Taking into account the variety of already known intransitive objects and systems, we pose the following problem. Based on information about the options A, ttf, and C separately, and information that A beats B and B beats C, can one conclude anything about superiority in the pair A-C? We discuss two possibilities.(1) Not only concrete decisions, but also a general algorithm for such inferences is possible.(2) A general normative model determining whether relations in various situations are (in)transitive is hardly possible. Decisions about transitivity/intransitivity are possible but inevitably context-dependent.

Why humans speed up when clapping in unison

Author(s): Lukeman, Ryan James (St. Francis Xavier University, Canada). *Contact:* `rlukeman@stfx.ca`.
Abstract: Humans clapping together in unison is a familiar and robust example of emergent synchrony. We find that in experiments, such groups (from two to a few hundred) always increase clapping frequency, and larger groups increase more quickly. Based on single-person experiments and modeling, an individual tendency to rush is ruled out as an explanation. Instead, an asymmetric sensitivity in aural interactions explains the frequency increase, whereby individuals correct more strongly to match neighbour claps that precede their own clap, than those that follow it. A simple conceptual coupled oscillator model based on this interaction recovers the main features observed in experiments, and shows that the collective frequency increase is driven by the small timing errors in individuals, and the resulting inter-individual interactions that occur to maintain unison.

Decision making 3 (Drummond West & Center)

Axioms and inference: a toolbox for abstract stochastic discrete choice

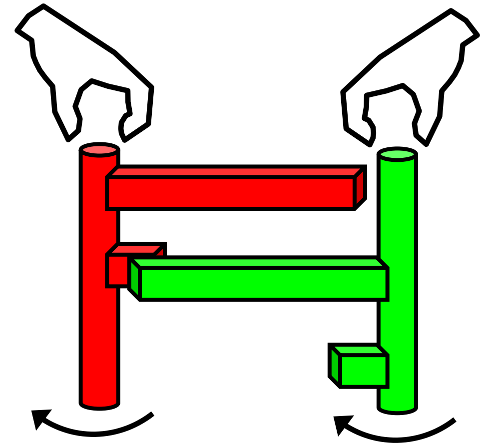
Author(s): McCausland, William James (University of Montreal, Canada). *Contact:* `william.j.mccausland@umontreal.ca`. *Abstract:* I describe and demonstrate an R package, providing tools for a research project whose purpose is to help us better understand the foundations of stochastic discrete choice. The toolbox includes datasets compiled from the context effects literature, the stochastic intransitivity literature, and from some recent experiments where we observe choices from all doubleton and larger subsets of some universe of objects. It provides graphical tools illustrating likelihood function and posterior density contours, as well as regions, in the space of choice probabilities, defined by various stochastic choice axioms, context effects and other conditions. Eventually, it will provide tools for parametric and non-parametric inference subject to various combinations of discrete choice axioms, as well as the testing of said axioms.

Distinguishing between contrast models of category generation

Author(s): Liew, Shi Xian[1]; Conaway, Nolan[2]; Kurtz, Kenneth J.[3]; Austerweil, Joseph L.[1] (1: University of Wisconsin - Madison; 2: Shutterstock; 3: Binghamton University). *Contact:* `liew2@wisc.edu`. *Abstract:* The generation of items in novel categories tends to be strongly influenced by how different they are to previously learned categories. We demonstrate how this idea of contrast can be meaningfully captured by two separate

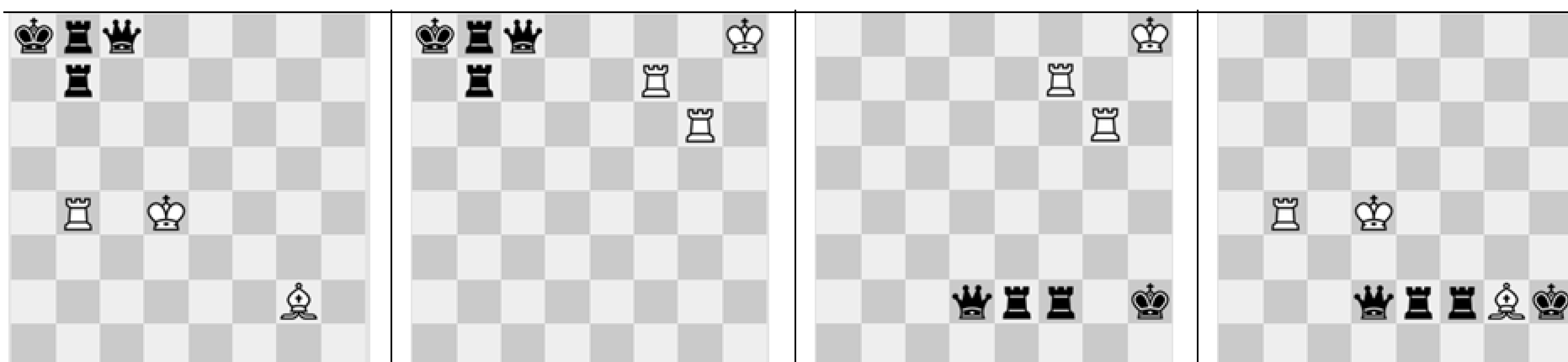
Presentation

Making decisions on intransitivity of superiority: is a general normative model possible?



Alexander Poddiakov

National Research University
Higher School of Economics

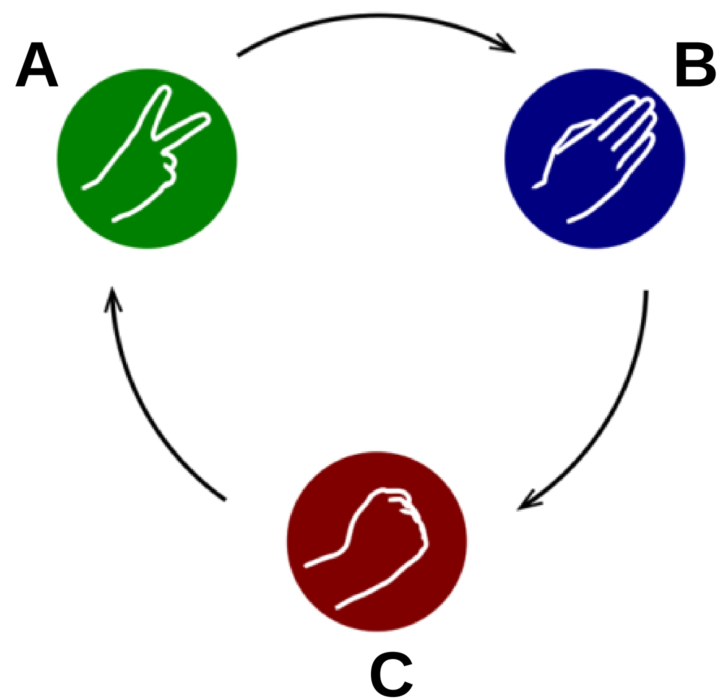


Intransitivity (intransitive cycle) of superiority:

$$A \succ B,\ B \succ C,\ C \succ A$$

where "$\succ$" means "dominates over", "is better than", "is preferable to" etc.

A popular analogy:
Rock-Paper-Scissors



https://en.wikipedia.org/wiki/Rock%E2%80%93paper%E2%80%93scissors#/media/File:Rock-paper-scissors.svg

This contrasts with transitivity of superiority (not with cyclic but with linear order):

$A \succ B$, $B \succ C$, $A \succ C$

A possible analogy: if $5>4$ and $4>3$ then $5>3$.

There are lots of transitive and intransitive relations in various areas. Many of these relations are trivial and not very interesting.

Yet relations of superiority (domination, preferability) and inferences about it seem so interesting and crucially important that a special axiom was introduced and accepted by many researchers.

The transitivity axiom:
if $A \succ B$ and $B \succ C$ then $A \succ C$

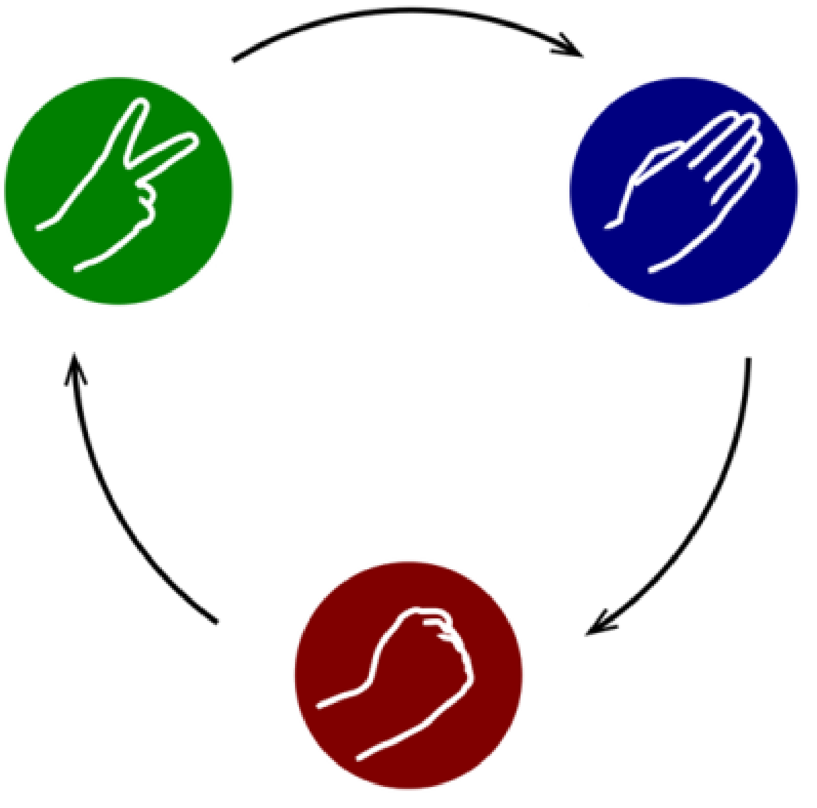
"If you have violated the transitivity axiom, … you are not instrumentally rational. The *content* of A, B, and C do not matter to the axiom".

(Five Minutes with K.E. Stanovich, R.F. West, and M.E. Toplak, 2016 https://mitpress.mit.edu/blog/five-minutes-keith-e-stanovich-richard-f-west-and-maggie-e-toplak)

If the transitivity axiom were universal and applicable everywhere, it would be very helpful for making decisions about superiority relations between A and C based on the data that: $A \succ B$ and $B \succ C$.

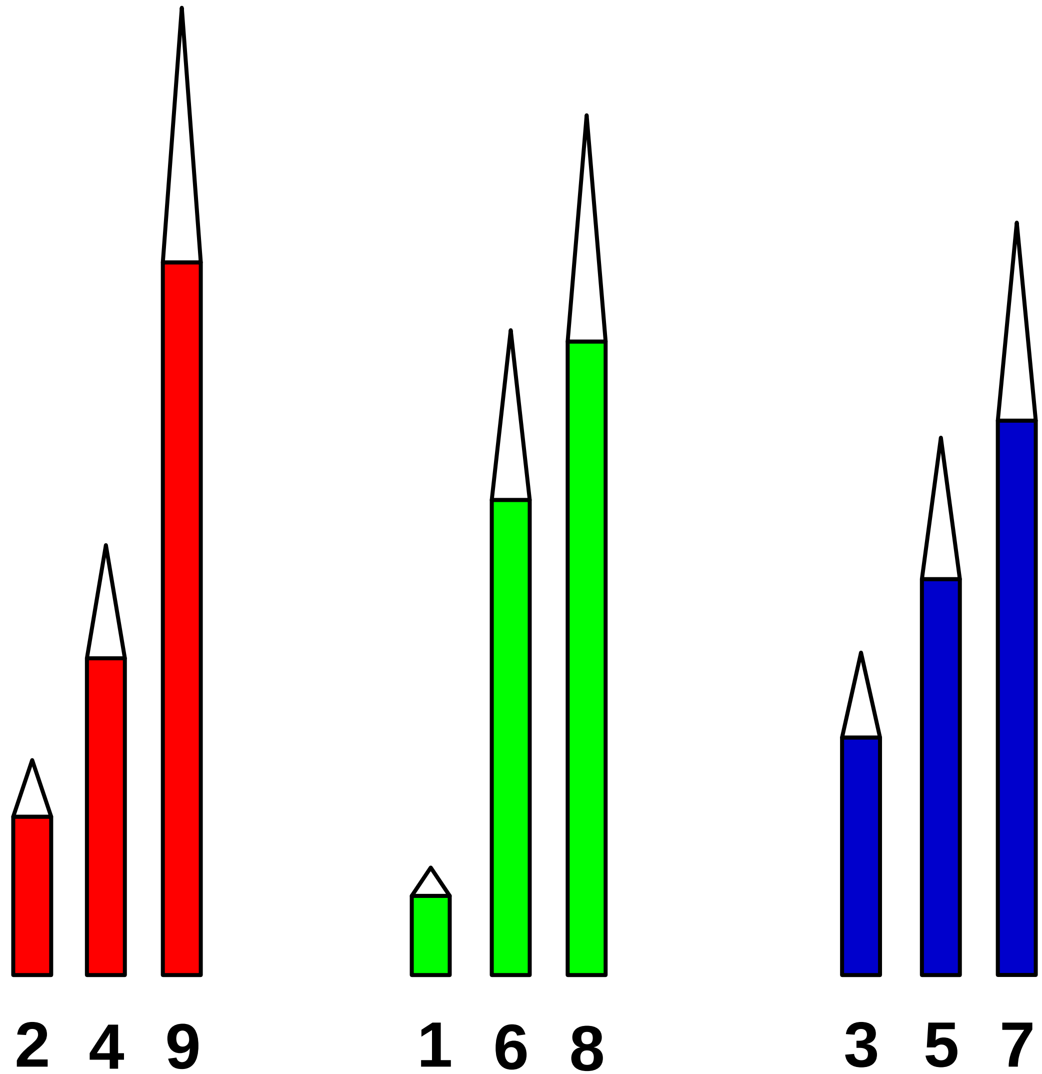
Yet…

We have a lot of math research on objects in intransitive relations (e.g. like in “rock-paper-scissors”)



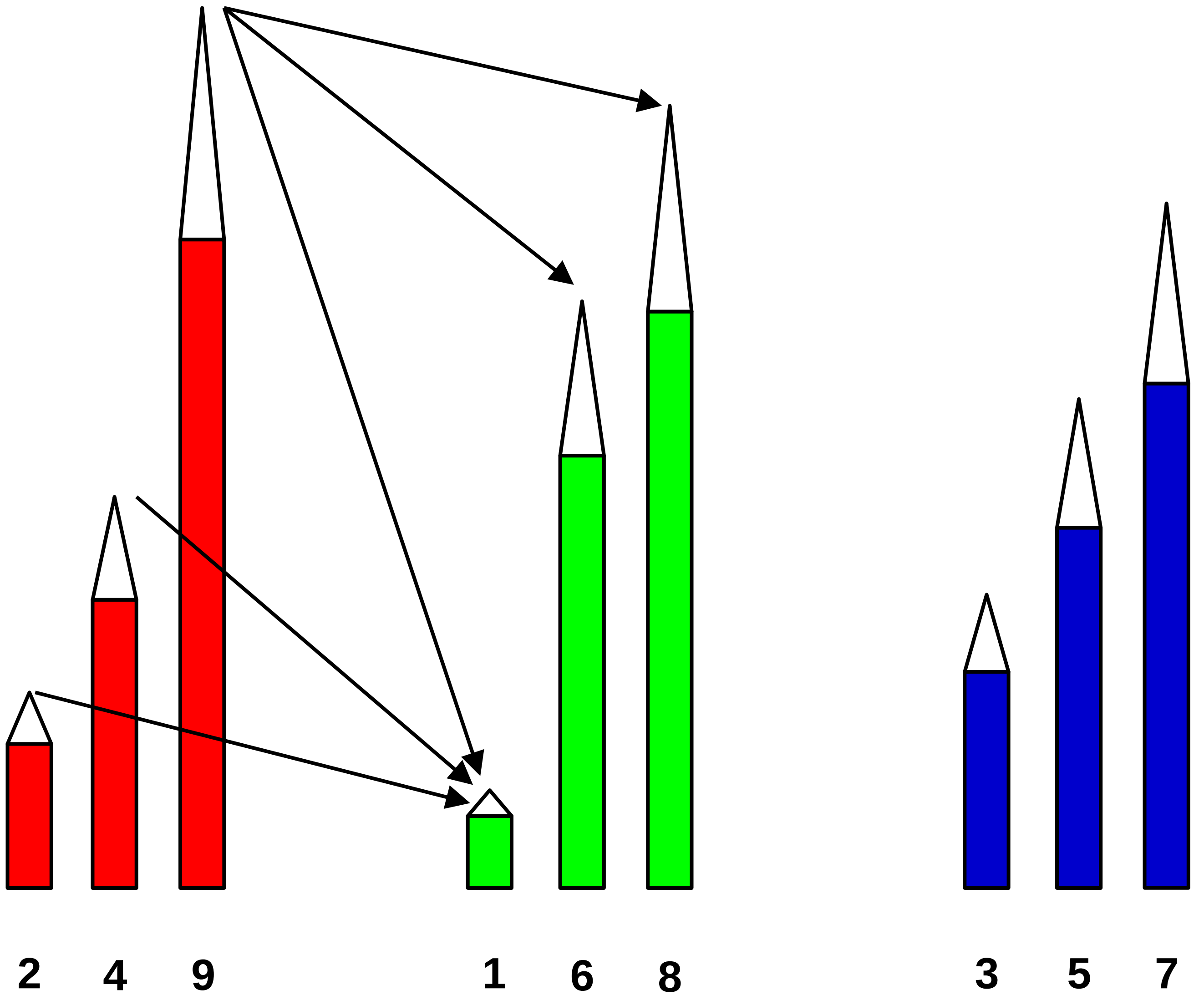
https://en.wikipedia.org/wiki/Rock%E2%80%93paper%E2%80%93scissors#/media/File:Rock-paper-scissors.svg

Let us consider 3 sets of 3 pencils of different lengths. We compare the length of each pencil with the lengths of all the other pencils.

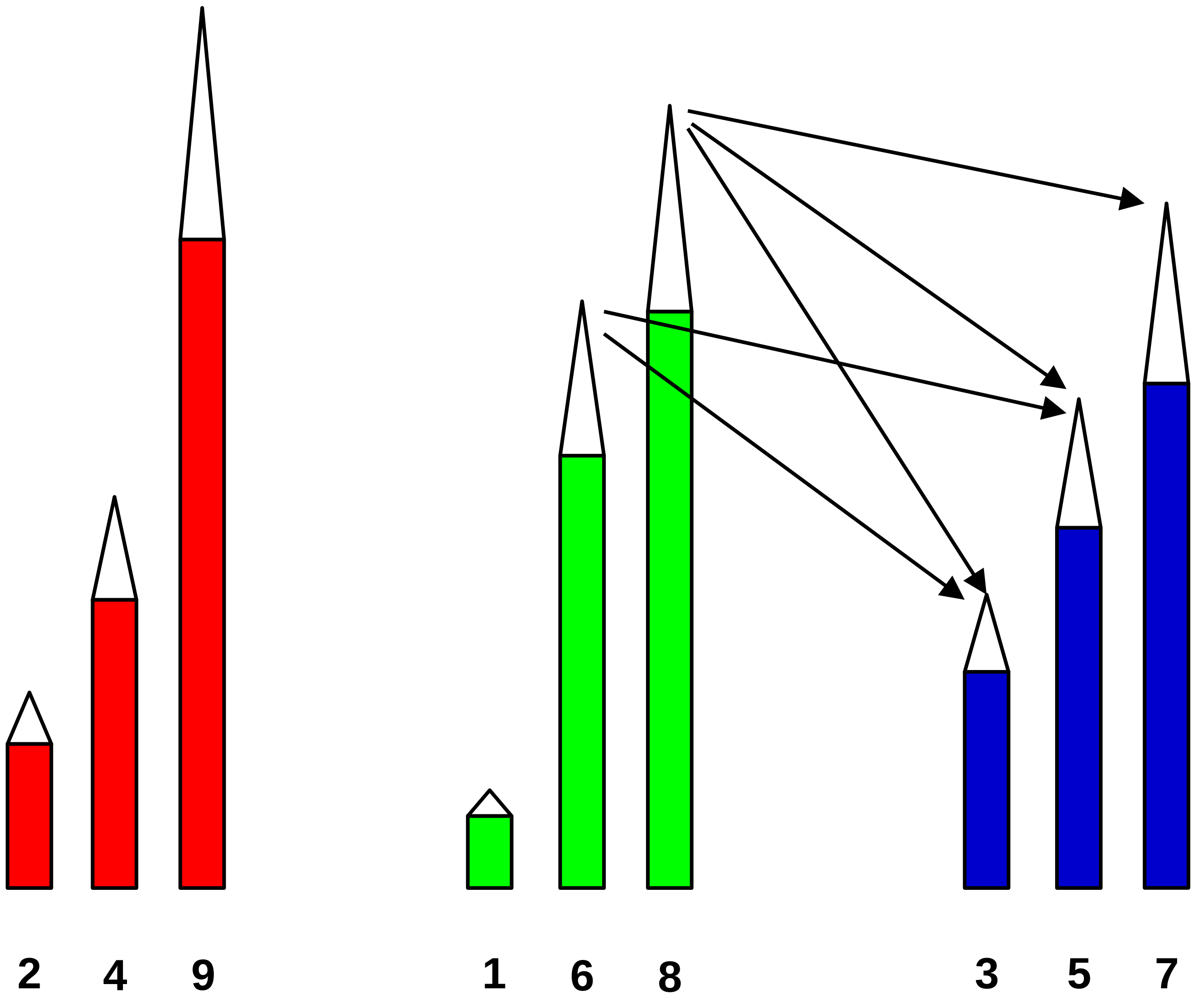


Numbers are taken from the "magic square" presented by Gardner (1974)

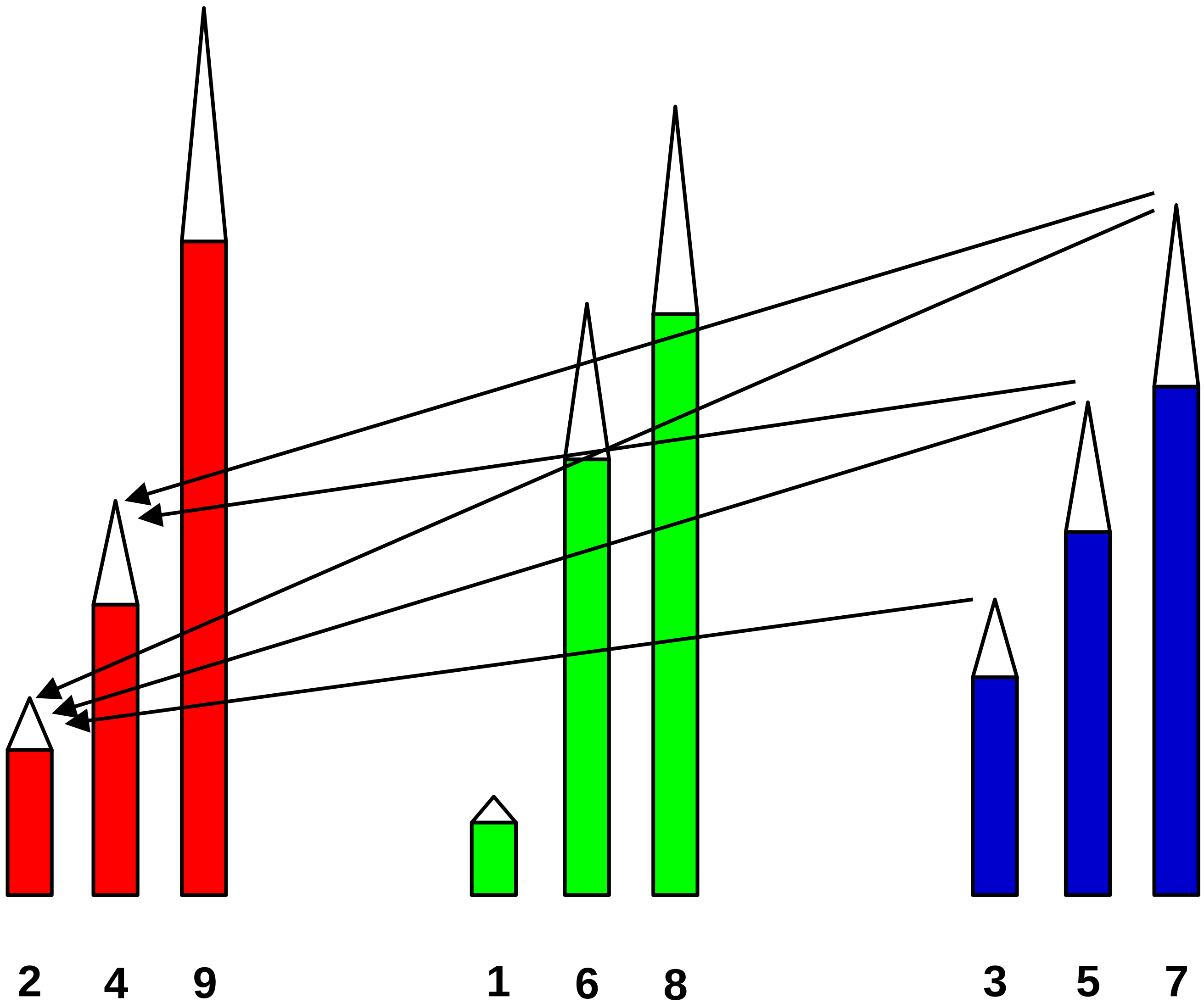
Red pencils beat green ones 5 out of 9 times



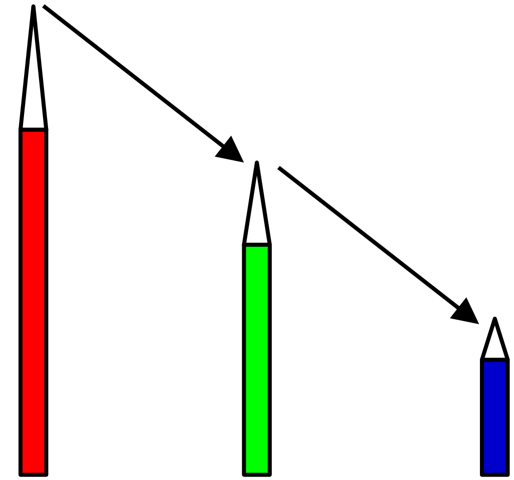
Green pencils beat blue ones 5 out of 9 times



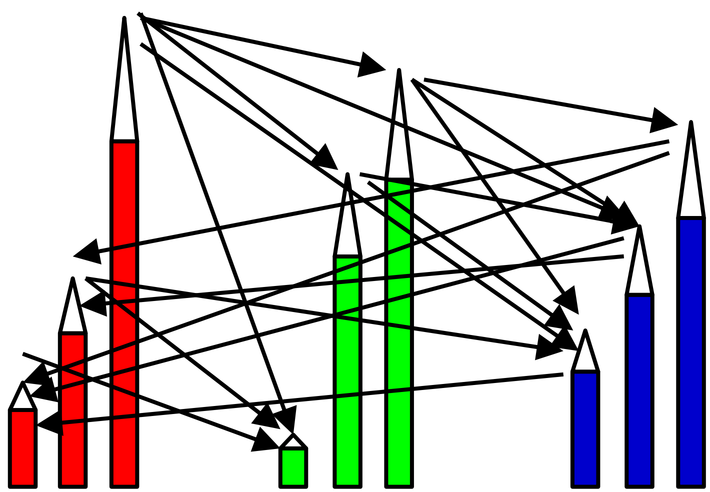
Blue pencils beat red ones 5 out of 9 times



Transitivity is bounded – it works in simple situations: during the comparison of 3 pencils – for the relation "to be longer"…



but it does not work in more complex situations: during the comparison of 3 sets of 3 pencils – for the relation "to often be longer"

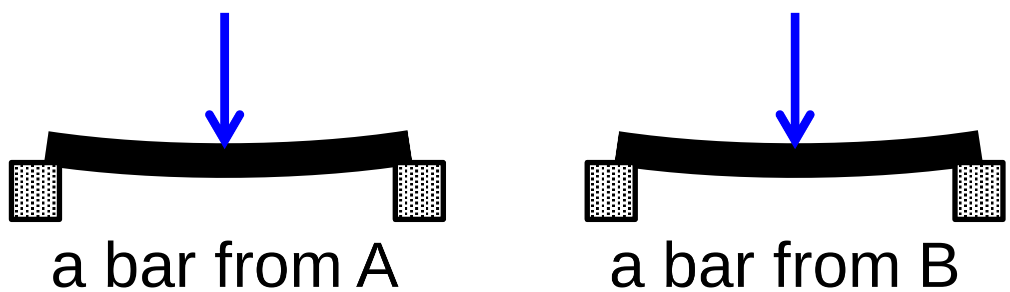


Only arrows from winning to loosing sets are shown

Consequences for the physical world: an example

Three factories A, B, and C produce iron bars.

We take bars from each factory and organize “a tournament between bars” - test the comparative strength between the bars in pairwise comparisons.

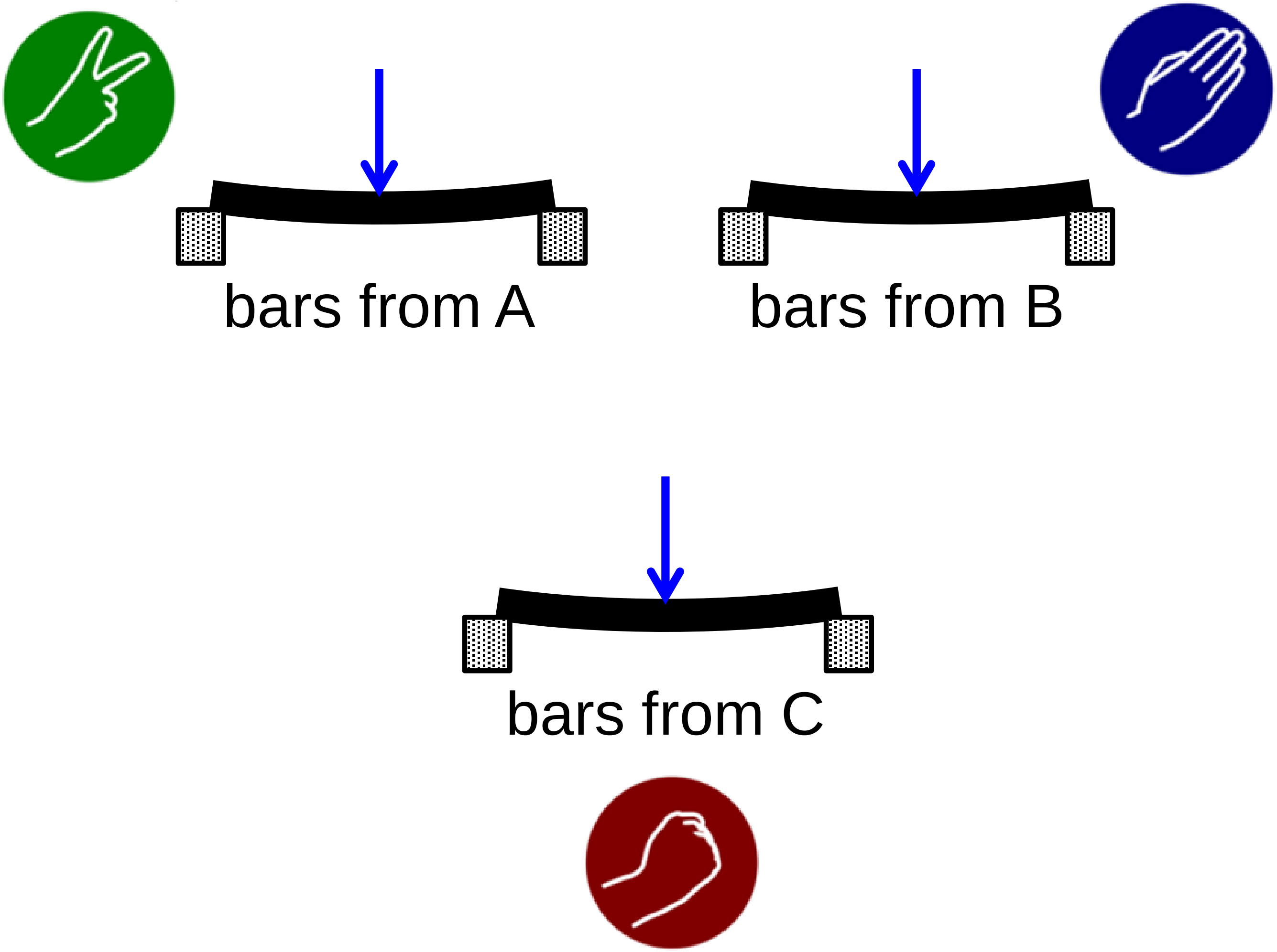


Is it possible that:

- bars from factory A are more often stronger than bars from B;
- bars from factory B are more often stronger than bars from C;
- bars from factory C are more often stronger than bars from A?

It is possible: Trybuła, S. (1961).
https://eudml.org/doc/264121

So, if making a bet, one should prefer A in pair A-B, B in pair B-C, and C in pair A-C.



Intransitive dice

National Museum of Mathematics, USA

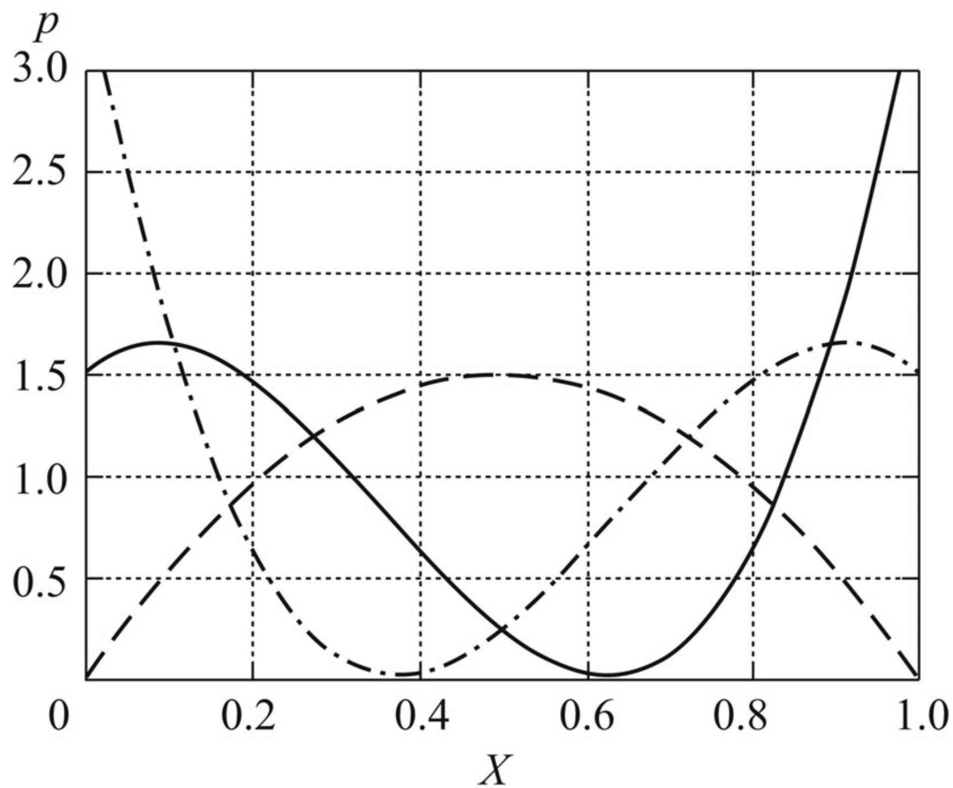
Purple (A): 4, 4, 4, 4, 0, 0
Yellow (B): 3, 3, 3, 3, 3, 3
Red (C): 6, 6, 2, 2, 2, 2
Green (D): 5, 5, 5, 1, 1, 1

A≻B, B≻C, C≻D, D≻A

Many extremely interesting results are obtained for various intransitive dice (including large sets of multi-sided dice), lotteries etc.

Lebedev, A. (2019): a study of intransitivity of 3 continuous random variables

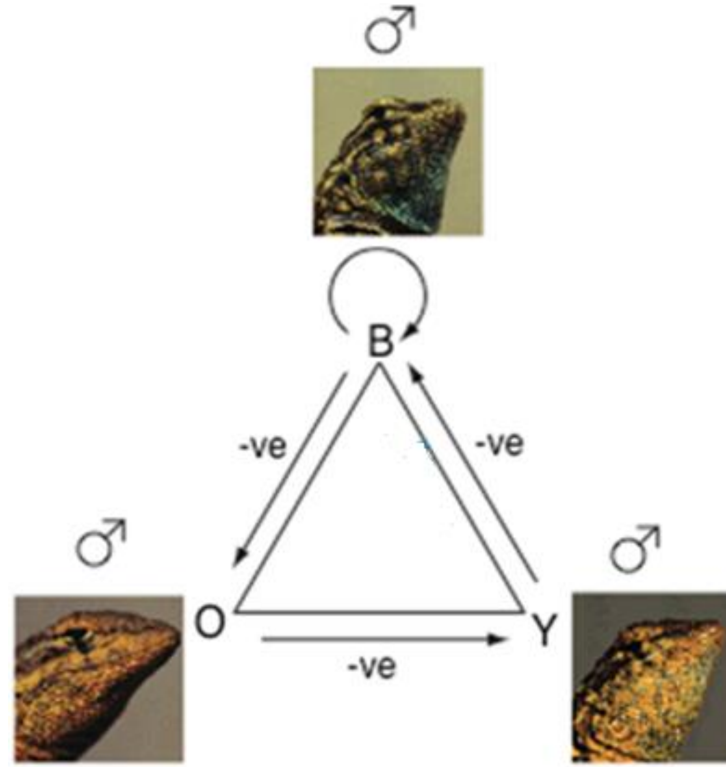
https://link.springer.com/article/10.1134%2FS0005117919060055



Some intransitive distributions of continuous random variables are possible and some are not. Intransitivity is not “recreational math”.

Biology

Intransitive competition - an important condition of biodiversity and co-existence



Sinervo, B., & Lively, C. (1996). The rock-paper-scissors game and the evolution of alternative male strategies. *Nature*, *380*.

Tens of articles on intransitive biological competition have been published in *Nature*.

So, not only the strength of the iron bars but also the biological competition of species and individuals can be intransitive.

“Robot Darwinism” in *BattleBots* shows (Atherton, 2013)

Geometry & Mechanics: The Intransitive Machines

I have designed geometrical and mechanical constructions in intransitive relations. From a mathematical point of view, it is **a new class of intransitive objects**. They show deterministic (not probabilistic) intransitive relations.

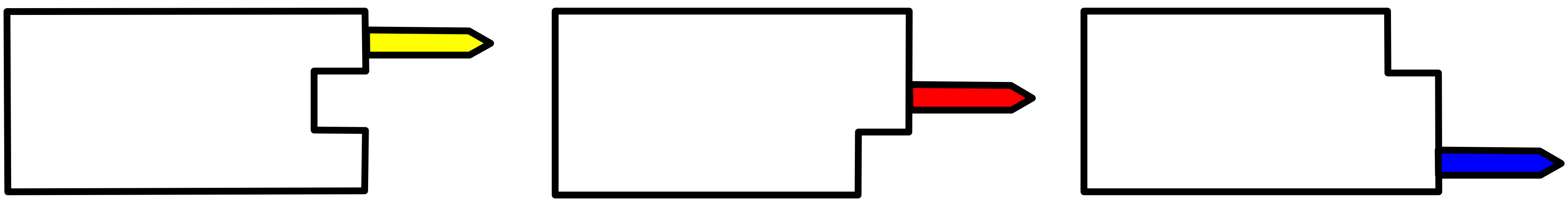
They are built as Condorcet-like compositions.

An Assyrian wheeled battering ram

https://commons.wikimedia.org/wiki/File:Assyrian_Attack_on_a_Town.jpg

Is it potentially possible that battering ram A can beat B, B can beat C and C can beat A?

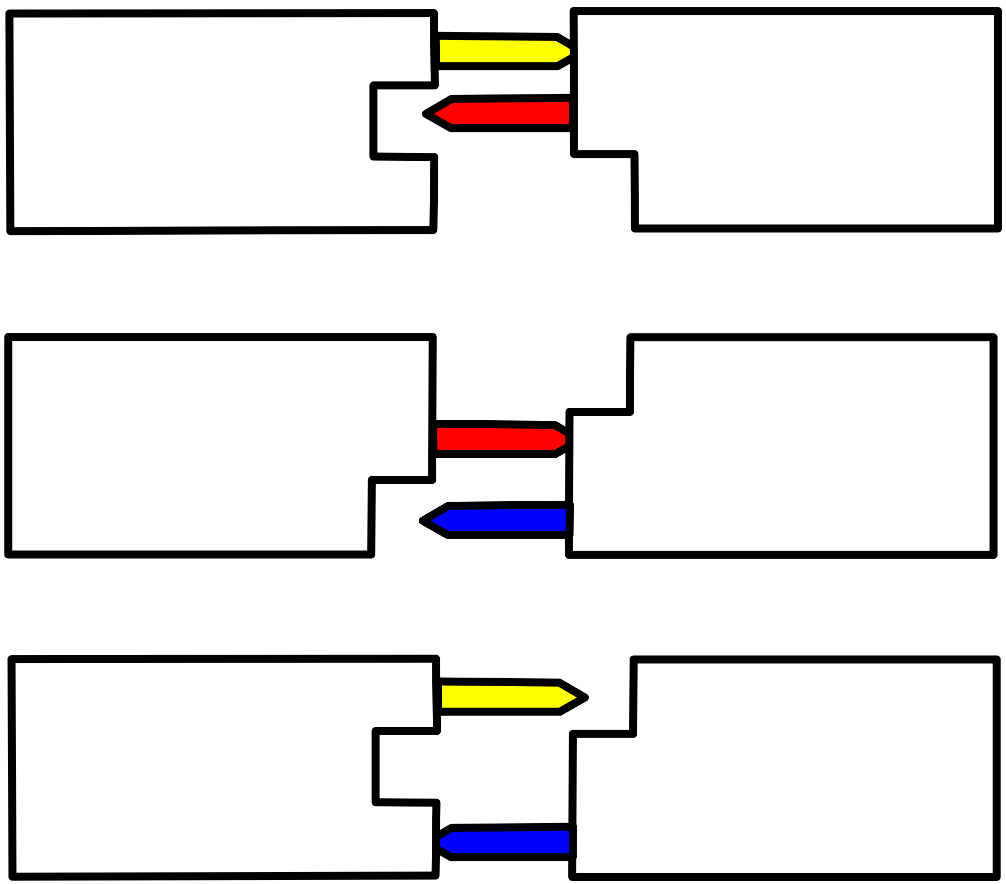
Intransitive Battering Rams
(Poddiakov, 2001, 2018)

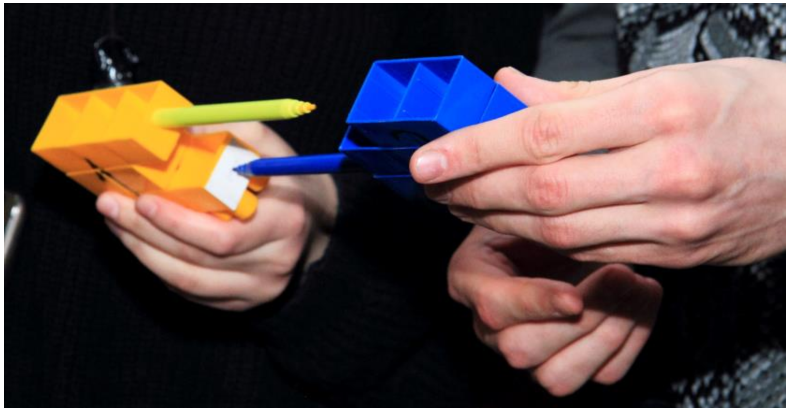


The Condorcet structure

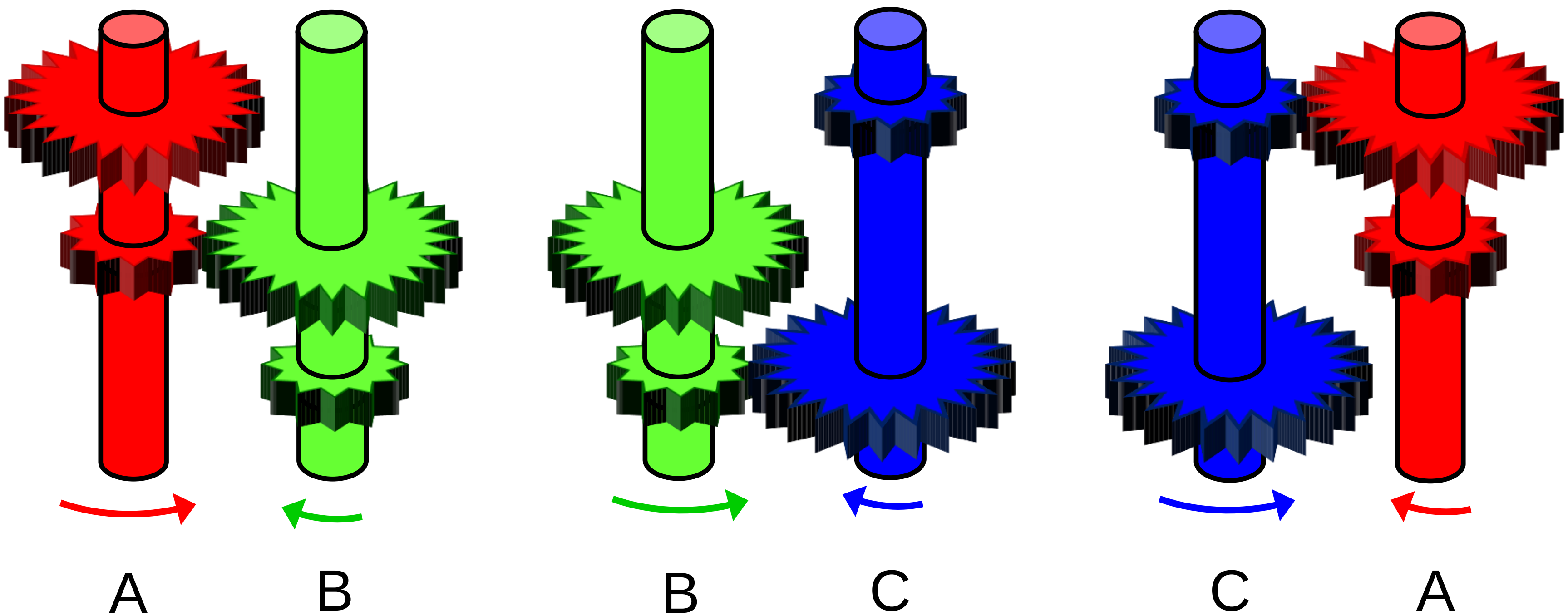
$$\begin{bmatrix} X & Y & Z \\ Z & X & Y \\ Y & Z & X \end{bmatrix}$$

Intransitive Battering Rams





Intransitive Double Gears



A rotates faster than B in pair A-B,
B rotates faster than C in pair B-C,
C rotates faster than A in pair A-C.

A more paradoxical and complicated version:
Oskar van Deventer's
Non-Transitive Gears-and-Ratchets

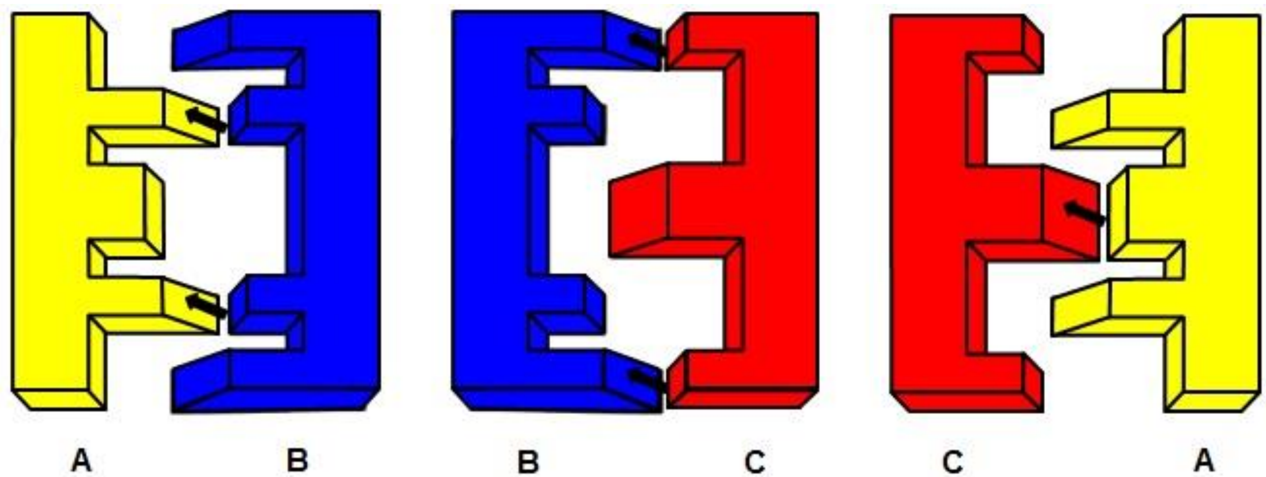
https://i.materialise.com/forum/t/non-transitive-gears-by-oskar/1167

Whatever element the first player chooses (a knob or a gear), the second player can always choose an element rotating faster than the element chosen by the first player.

https://i.materialise.com/forum/t/non-transitive-gears-by-oskar/1167

A huge number of such geometrical Condorcet-like compositions is possible.

Intransitive Ramps



A lifts B
B lifts C
C lifts A

Psychological studies of people’s beliefs about the possibility of intransitive relations in various areas

(Poddiakov, 2010, 2011; Bykova, 2018, under Poddiakov’s supervision)

Method: Participants are asked if certain objects and relations are potentially possible or not.

Participants: 169 people (17-28 yrs, 121 females, 48 males).

Item A

There are 3 teams with 6 wrestlers in each team. During their tournament, each wrestler of one team meets and wrestles with each wrestler from two other teams.

It is known that the wrestlers of the 1st team beat the wrestlers of the 2nd team more often than they are beaten by them, and the wrestlers of the 2nd team beat the wrestlers of the 3rd team more often than they are beaten by them.

Is it possible that the wrestlers of the 3rd team beat the wrestlers of the 1st team more often than they are beaten by them?

Item A

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Is it possible that the wrestlers of the 3rd team beat the wrestlers of the 1st team more often than they are beaten by them?

Results: 81% of the participants gave the right answer - "yes, it's possible"

Item B

There are 3 boxes with 6 pencils of different lengths in each box. We compare the length of each pencil with the length of all the other pencils. We learn that the pencils from the 1st box are often longer than the pencils from the 2nd box, and the pencils from the 2nd box are often longer than the pencils from the 3rd box.

Is it possible that the pencils from the 3rd box are often longer than the pencils from the 1st box?

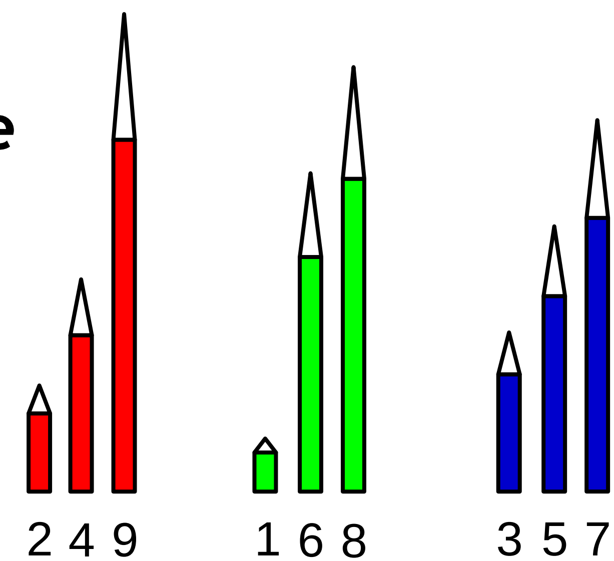
Item B

There are 3 boxes with 6 pencils of different lengths in each box. We compare the length of each pencil with the length of all the other pencils. We learn that the pencils from the 1st box are often longer than the pencils from the 2nd box, and the pencils from the 2nd box are often longer than the pencils from the 3rd box.

Is it possible that the pencils from the 3rd box are often longer than the pencils from the 1st box?

Results: only 29% **of the participants gave the right answer - "yes, it's possible"**

(the simpler example of intransitive pencil sets is on the right)



!!!

Problems “3 teams with 6 wrestlers in each team” and “3 boxes with 6 pencils in each box” are **of the same logical structure**.

But, paradoxically, the most participants (81%) believe in the intransitivity of the wrestlers’ teams, and only 29% believe in the the intransitivity of length in the pencils’ boxes.

Why? We have not studied it yet. Only some hypotheses can be formulated.

Similarly, most people (90%) believe in the potential possibility of intransitive battering rams and only 20% believe in intransitive double-gears.

They believe in intransitive competition of microorganisms (95%) and, to a lesser extent, in intransitive competition between chess computers (60%).

And so on.

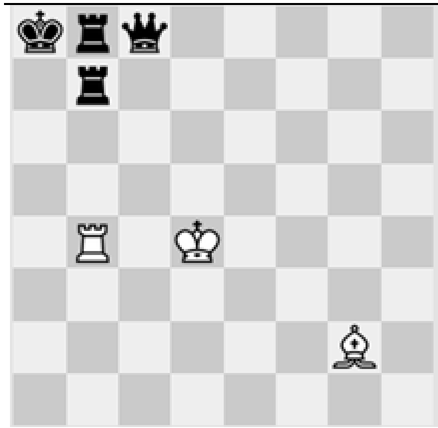
Thus, beliefs about intransitivity of superiority are domain-specific. The participants believe that intransitive relations exist in some domains but not in others (though really they are possible in the latter domains as well).

Intransitive materials again - a new fact for game theory:

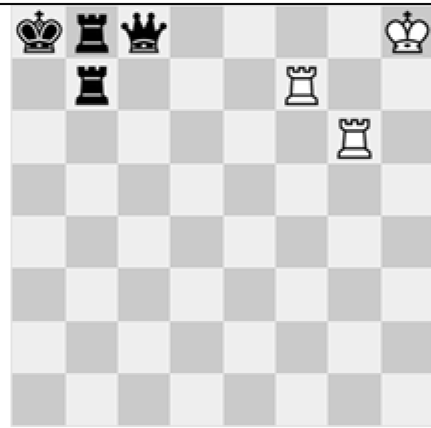
Intransitive positions in chess and checkers

Intransitive chess positions
(Poddiakov, 2016)

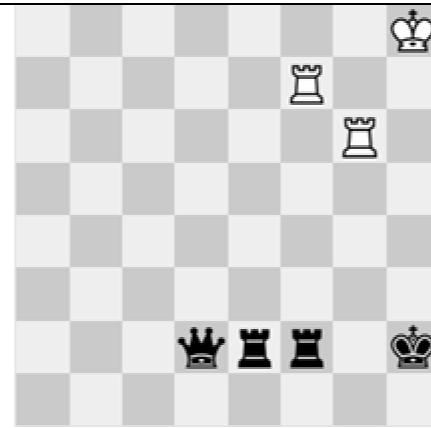
White starts in all the positions



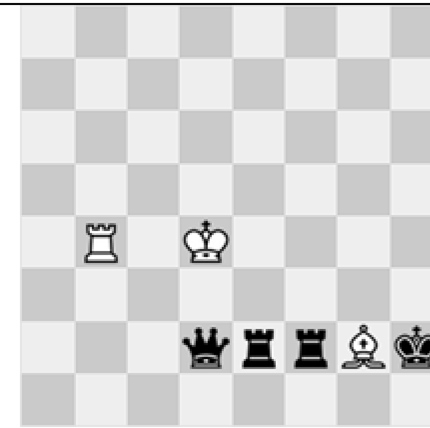
Pos. A(white)≻
Pos. B(black)



Pos. B(black)≻
Pos. C(white)

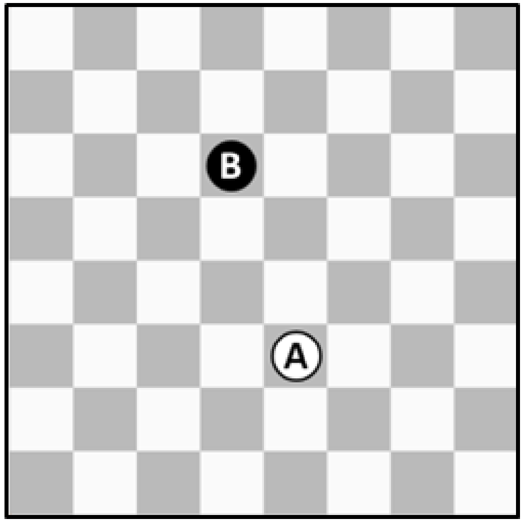


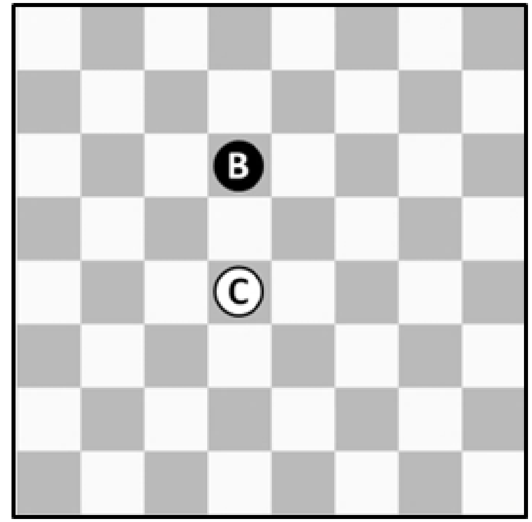
Pos. C(white)≻
Pos. D(black)

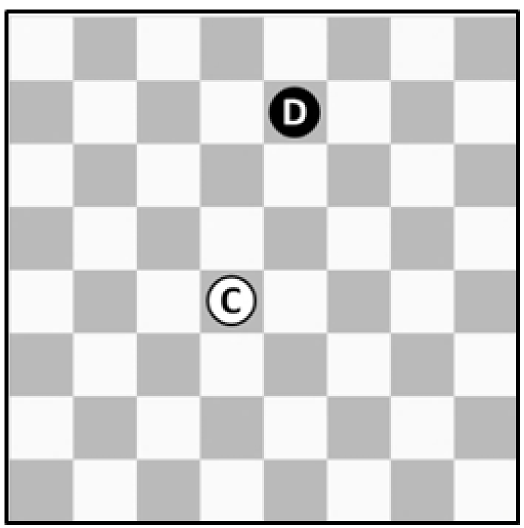


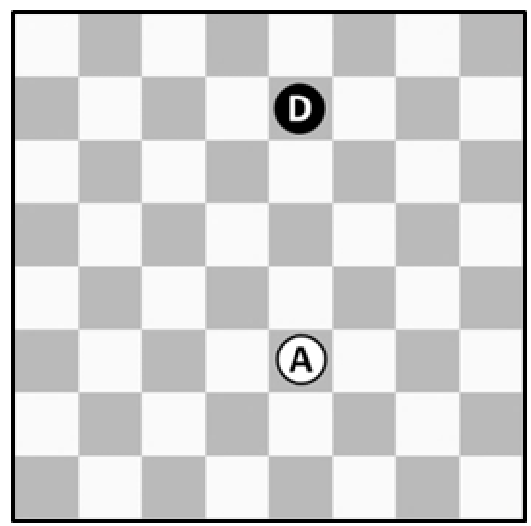
Pos. D(black)≻
Pos. A(white)

Intransitive checkers positions
(Zhurakhovsky, 2017)



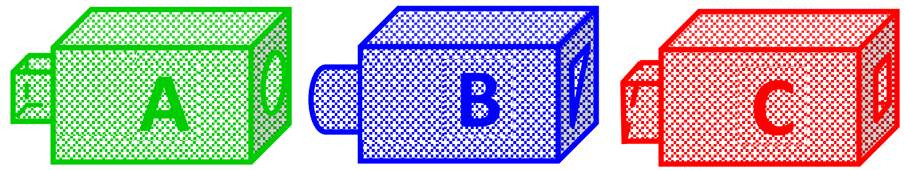


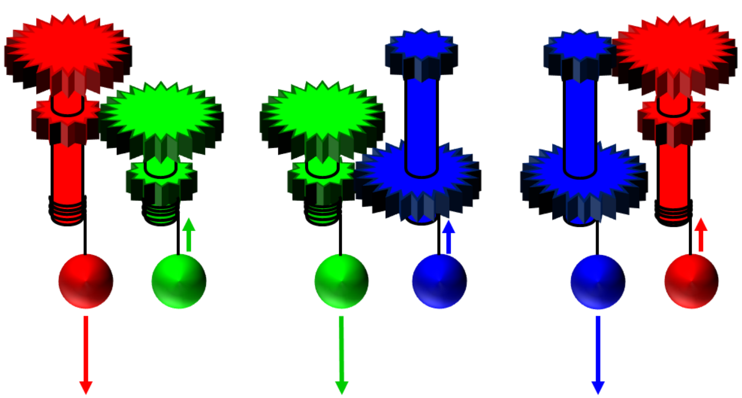


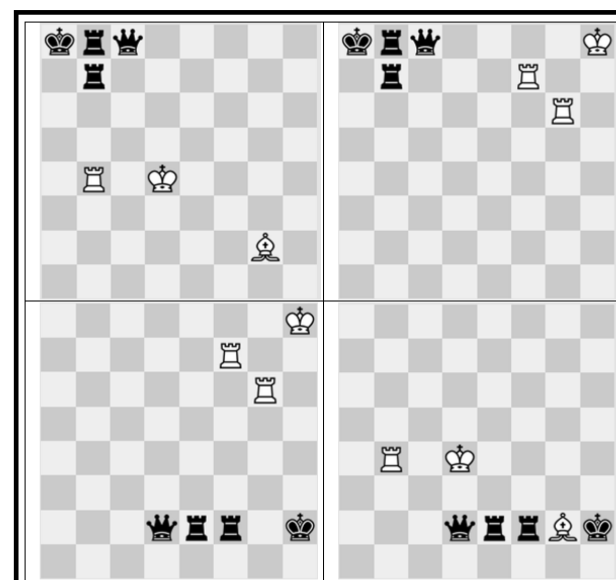


$A \succ B$
$B \succ C$
$C \succ D$
$D \succ A$

Thus, a great variety of different objects and systems in intransitive relations does exist.







So, the key statement in the area of transitivity/intransitivity should be more multi-dimensional than “if you have violated the transitivity axiom, you are not rational”.

It seems reasonable to distinguish between four types of situations (Poddiakov, 2010).

(1) Relations are objectively transitive and problem solvers make correct conclusions about their transitivity.

(2) Relations are objectively transitive, but problem solvers wrongly consider them as intransitive.
Most cognitive psychological studies are conducted in this paradigm.

(3) Relations are objectively intransitive and problem solvers make correct conclusions about their intransitivity (e.g., intransitivity of intransitive dice, lotteries etc.).

(4) Relations are objectively intransitive, but problem solvers wrongly consider them as transitive (e.g. because they take the transitivity axiom for granted). Surprisingly, this type has been minimally studied by cognitive psychology.

Taking into account the variety of already known intransitive objects and systems, one can pose the following problem.

Based on information about the options A, B, and C separately, and information that A beats B and B beats C can one conclude anything about superiority in the pair A-C?

Two possible answers

1) Not only concrete decisions, but also a general algorithm for such inferences is possible.

2) A general normative model determining if relations in various situations are (in)transitive is hardly possible. Decisions about transitivity/ intransitivity are possible but inevitably context-dependent (content of A, B, and C does matter).

Even for dice, “the information that A beats B and B beats C has almost no effect on the probability that A beats C” (Gowers, 2018, https://gowers.files.wordpress.com/2017/07/polymath131.pdf; see also Conrey et al., 2016, https://www.tandfonline.com/doi/abs/10.4169/math.mag.89.2.133).

The same seems correct for higher levels of complexity but must be proven.

Perhaps the next step - which is my dream - is to prove algorithmic undecidability of the problem of determining transitivity/intransitivity in various situations.

(“An undecidable problem is a decision problem for which it is proved to be impossible to construct an algorithm that always leads to a correct yes-or-no answer”, https://en.wikipedia.org/wiki/Undecidable_problem).

Two quotations as conclusion

T. S. Roberts (2004):

"Transitivity and intransitivity are fascinating concepts that relate both to mathematics and to the real world we live in".

P. Fishburn (1991):

Rejection of intransitivity is analogous with "rejection of non-Euclidean geometry in physics would have kept the familiar and simpler Newtonian mechanics in place, but that was not to be".

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