PRAAG-2018

Alexander Perepechko, Moscow Institute of Physics and Technology

Automorphism groups of affine varieties consisting of algebraic elements

Abstract: This talk is based on joint works with S. Kovalenko, A. Regeta, and M. Zaidenberg, [KPZ], [PR]. Given an affine variety X, an automorphism $g \in \mathrm{Aut}(X)$ is called *algebraic* if it is contained in an algebraic subgroup of $\mathrm{Aut}(X)$. We conjecture that the following conditions on the neutral component $\mathrm{Aut}^\circ(X)$ are equivalent:

(1) it consists of algebraic elements;
(2) it is exhausted by an inductive limit of algebraic subgroups;
(3) it is a semidirect product of an algebraic torus and an abelian unipotent group;
(4) its tangent algebra consists of locally finite elements;
(5) its unipotent elements comprise an abelian subgroup.

In the case $\dim X \leq 2$, their equivalence follows from [KPZ]. In the case of higher dimensions, we prove some relations between them using the following result.

Theorem. [P.–Regeta] *Let X be an affine algebraic variety over an algebraically closed field* $\mathbf{k}$ *of zero characteristic, $\mathcal{U}(X) \subset \mathrm{Aut}(X)$ be a subgroup generated by unipotent elements, and let $\mathrm{Der}^{lf}(\mathbf{k}[X]) \subset \mathrm{Der}(\mathbf{k}[X])$ be a subset of locally finite derivations. Then $\mathcal{U}(X)$ is abelian if and only if $\mathrm{Der}^{lf}(\mathbf{k}[X])$ is an algebra. In such a case $\mathrm{Der}^{lf}(\mathbf{k}[X])$ is equal as a vector space to a direct sum of abelian subalgebras $\mathfrak{h}$ and $\mathfrak{u}$, where $\mathfrak{h}$ consists of semisimple derivations and $\mathfrak{u}$ consists of locally nilpotent derivations.*

[KPZ] S. Kovalenko, A. Perepechko, and M. Zaidenberg. *On automorphism groups of affine surfaces*, to appear in: Algebraic Varieties and Automorphism Groups, Advanced Studies in Pure Mathematics **99**, pp. 207-286.

[PR] A. Perepechko, A. Regeta. *Automorphism groups of affine varieties with only algebraic elements*, in preparation.